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POINT AND PATH PERFORMANCE OF LIGHT AIRCRAFT

A Review and Analysis

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16. Abstract

The literature on methods for predicting the performance of light aircraft is reviewed. The methods discussed in the review extend from the classical instantaneous maximum or minimum technique to techniques for generating mathematically optimum flight paths. Classical point performance techniques are shown to be adequate in many cases but their accuracies are compromised by the need to use simple lift, drag, and thrust relations in order to get closed form solutions. Also the investigation of the effect of changes in weight, altitude, configuration, etc. involves many essentially repetative calculations. Accordingly, computer programs are provided which can fit arbitrary drag polars and power curves with very high precision and which can then use the resulting fits to compute the performance under the assumption that the aircraft is not accelerating. Path performance programs are also provided which permit the user to specify the variations with time of any two of six quantities (V, h, P, α, W, γ) and to receive as output the correct variations with time of the other four quantities. This program is desirable when optimum performance is not obtained under steady state conditions but rather during what may be termed maneuvers. Detailed program listings and instructions for use are provided as are several worked out example problems. Programs to compute mathematically optimal flight paths are not provided because those cases where solutions have been obtained lack generality and because considerable mathematical sophistication is required to determine which problems can be treated by existing techniques. Also, the path performance program can be run repeatedly to identify quasi-optimal paths.

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GENERAL INTRODUCTION

It has long been recognized by light aircraft manufacturers that the technical consideration which exerts the largest influence on sales is the performance of the aircraft. If an aircraft can climb higher, fly further, cruise faster, and land and take off on shorter runways than other aircraft for the same payload and price, then it will usually sell better than its competition. A substantial portion of the engineering effort expended on a new design is therefore devoted to estimating the performance improvement resulting from a given change in configuration or powerplant. What one would like to be able to do is to suggest those changes which result in optimum performance for the price.

The National Aeronautics and Space Administration undertook the present study to assist the light aircraft industry and aeronautical education in general. It was felt that the task of selecting the best performing configuration for the price would be greatly facilitated if the pertinent research results of the last 35 years were readily at hand in easily-usable forms. A computer program employing a collection and arrangement of those methods and data most applicable to the estimation of light aircraft performance would appear to satisfy these requirements. The present work seeks to provide these programs along with a detailed review of the methods used and some worked-out examples. The work is thus analogous to previous studies (Refs. 1, 2) which approached the prediction of riding and handling qualities of light aircraft in a similar fashion.

As will become evident from the literature review following, performance estimation can be treated at three levels of sophistication. The first, which may be termed static or point performance, is concerned with the maximum values for certain parameters such as speed, rate of climb, etc. assuming that nothing changes with time. The equations expressing the power required for level flight as a function of speed and altitude are algebraic and therefore fairly easy to evaluate. The variation in maximum power available as a function of flight speed and altitude can be evaluated point by point, if not analytically. The region between these two data sets on the speed-power plane is that for which steady flight is possible. Thus, by finding intersections of the two functions one has maximum and minimum speed while the maximum difference between the two curves is a measure of the maximum rate of climb. Other preformance parameters are calculated with similar directness. The calculations are usually performed graphically because an analytical solution requires the extraction of the roots of a fourth or higher order polynominal, a laborious procedure if done by hand. It will be recognized also that determining the effect of a change in configuration or power plant involves many calculations if one wishes to see the effect at all weights and operating altitudes.

Until about 20 years ago all aircraft manufacturers used these performance estimation techniques which were first developed in the early 1930's. The methods are generally as reliable as the quality of the input lift, drag, and thrust data. The results are easily interpreted and can be checked in flight through appropriate tests. With the advent of the modern digital computer, the major airframe manufacturers began to give consideration to more sophisticated means of describing the manner in which an airplane performs. They recognized for example that the time required to reach a given altitude could be minimized by varying the speed as the altitude increases and that the range could be increased on some aircraft by allowing the altitude to increase as the fuel is burned. In other words, the path over which the aircraft flies determines the performance of the vehicle. Hence they began to integrate the differential equations which describe aircraft motion with various types of control inputs to see what paths are produced. The digital computer permits one to investigate a large number of cases quickly and relatively inexpensively. Through a trial and error process one can get a good indication of how to fly a particular mission to obtain optimum results.

This level of sophistication is in common use among the large airframe constructors today. It has long been recognized, however, that an optimum path obtained in this fashion cannot be shown to be an optimum in the mathematical sense. The development of mathematically optimum flight paths has been a subject of theoretical research for at least a hundred years. Solutions of several simple problems have been obtained but a general procedure that is successful in a large number of cases has thus far eluded formulation.

The purpose of this report is to critically review methods available for estimating most aspects of light aircraft performance at all three levels of sophistication and to render those methods which are regarded as most accurate into fast, easy-to-use forms employing a digital computer. Through this device it is hoped that light aircraft designers can investigate a wider range of parameters economically in their search for improved performance in their vehicles. The programs and explanations are written at such a level that they should be readily intelligible to recent B.S. graduates.

As written here, the vehicle lift, drag, and thrust terms in the performance equations are represented by implicit functions. To obtain numerical solutions, explicit functions are required. These the program obtains by making rather general fits of user-supplied data. Unfortunately, it was not possible within the scope of the present work to eliminate the requirement for the user to supply these data. It would have been desirable to ask the user to specify only the aircraft geometry and the power plant and propeller characteristics and to have the program compute the lift, drag, and thrust characteristics needed for the performance computation.

The work begins with a review of the pertinent literature of the past 40 years. Estimation techniques based on the point performance concept are then developed. These techniques have been programmed for computer solution. The use of this program is then explained and some sample results for a typical light aircraft are given.

The next section treats the path performance concept. Again, an easily-used computer program has been developed to perform the computations. Its use and basis are explained and typical results are provided.

An appendix provides a detailed derivation of the equations for path performance while listings of the Fortran IV programs used to compute point and path performance are given in two additional appendices.

Other appendices present programs for fitting power curves, lift-drag curves, the basis for the integration technique used on the path performance equations, and a more detailed discussion of the nature of the fuel-flow-power relationship in piston engines.

The reader will perhaps note the absence of a reference to the standard text by Perkins and Hage (John Wiley 1949) and the failure to follow the nomenclature of this text which is by now fairly standard. However, it seemed that because the equations selected for computer solution are really simplifications of those used in stability analysis, the notation should follow that common in stability analysis. Some modifications in this view were found to be necessary in order to accomodate the more general drag polar used in the present work. It is hoped that these departures from common usage will not prove too disconcerting.

LITERATURE REVIEW

Point Performance

"General Formulas and Charts for the Calculation of Airplane Performance", TR-408, by Oswald (Ref. 3) and "General Airplane Performance", TR-654, by Rockefeller (Ref. 4), published in 1932 and 1939 respectively, represent the state of the art in the prediction of point performance. Oswald's work presents a series of performance charts for airplanes equipped with modern unsupercharged engines and fixed-pitch metal propellers; these charts yield the performance characteristics (maximum level flight speed, maximum rate of climb, service ceiling, absolute ceiling, etc.) as a function of the parasite drag loading, effective span loading, and thrust horsepower loading. Oswald later extended his analysis to include the case of supercharged engines (Ref. 5) while White and Martin (Ref. 6) made a similar analysis for the case of constant-speed propellers with no supercharging. In each of the analyses mentioned above special assumptions were made regarding the variation of engine power with altitude and engine speed and the variation of propulsive efficiency with altitude and air speed. These assumptions along with the assumption of a parabolic drag polar are necessary to obtain a problem which is tractable by hand solution techniques or in closed form.

Rockefeller decided that with new engine and propeller developments it would be desirable to attack the problem in a more general manner in order to obtain a method of performance calculation basically independent of the particular engine-propeller combination but readily adaptive to any type. Thus, he developed the equations for the analysis of the performance of an ideal airplane--an airplane for which the thrust power is independent of speed, the parasite drag is constant, and the lift coefficient has an infinite maximum value--in order that the charts developed for use in practical calculations would for the most part apply to any type of engine-propeller combination and system of control, the only additional material required consisting of the actual engine and propeller curves for the propulsion unit. Rockefeller also presented his results graphically as performance charts.

Accurate prediction of point performance characteristics requires reliable information on the power the aircraft can put into the airstream. For propeller-driven aircraft NACA TR-640 (Ref. 7) and WR L-286 (Ref. 8) present propeller data obtained from aerodynamic wind tunnel tests. The data is presented as a series of four design charts for each propeller tested; these charts have been the standard NASA format since 1929 (see Appendix F, Figure (F-1) for an example). Although its basic intent was to reveal the effects of changes in solidity resulting either from increasing the number of blades or from increasing the blade width, TR-640 is probably more widely known for its outline of the procedures required to compute the propeller thrust from the propeller design charts. A step-by-step procedure for calculating the power available is given in Appendix F of the present work along with a set of propeller design charts for the R.A.F. 6 two blade propeller.

The value of knowing the static performance characteristics is voiced by Thompson in Reference 9. He mentioned that one of the most perplexing guessing games in cross country flying is choosing the most favorable altitude and true airspeed for cruising flight. As a means of solving the cruising dilemma for level flight with a light airplane, normally operating engine, and constant speed propeller he suggested that:

(1) The high speed dash should be made at near sea level at maximum power.

(2) Normal cruising at 65-75% power should be made at the highest altitude at which these powers are available using full throttle and normal cruising RPM.

(3) Maximum range airspeed should be 1.4 to 2.0 times the flaps up stall speed depending on aerodynamic cleanness.

(4) Range is independent of altitude if airspeed is maintained at correct best range speed for each altitude.

(5) For best range at higher airspeeds, the optimum altitude is progressively higher.

(6) In moderate headwinds, the speed for maximum range should be increased about 10%.

(7) For maximum endurance, the airplane should be flown between 20 and 30 percent above flaps up stall speed, depending upon where minimum power is required to sustain level flight.

The suggestions given by Thompson are generally in good agreement with the results obtained from a point performance analysis of the Cessna 182 (see the section on Examples of Point Performance Calculation). Similar agreement was also found using a path performance analysis when flying near the angle of attack for best lift to drag ratio. These analyses were made using the point and path performance programs presented in Appendices C and D respectively.

In recent years new interest has arisen in improving the performance of light aircraft. As noted in Reference 10 the basic technology and configurations of most of the present light airplane fleet were developed before the advent of the high speed computer, jet transport, high lift technology, advanced stability and control analysis methods, analytical descriptions of handling qualities, and greatly improved wind tunnel testing techniques. Since this advanced technology has not been widely applied to light aircraft, they have not kept pace with the improvements achieved by commercial airliners. Roskam and Kohlman found by parametric variation that aerodynamic design modifications can be made to improve significantly the performance of light aircraft. They used a relatively simple computer program to evaluate the speed for best range, maximum level flight speed, specific range, maximum rate of climb, and speed for maximum rate of climb in terms of the predicted lift and drag coefficients resulting from specific geometric modifications.

Accurate prediction of static performance requires good estimates of the lift and drag coefficients as a starting point. An ideal procedure for obtaining suitable values of these coefficients would require that one specify only the body coordinates, speed, and altitude of the airplane to obtain in a precise fashion both the lift and drag coefficients as functions of angle of attack; unfortunately, such a procedure is not as yet available. Historically, the drag coefficient has been much more difficult to estimate accurately than the lift coefficient. Reference 11 is an example of a sophisticated method for obtaining aerodynamic characteristics of multi-component airfoils--airfoils with leading or trailing edge high lift devices--in subsonic viscous flows. The calculated aerodynamic characteristics include pressure distribution, lift, pitching-moment, and skin friction drag up to incipent separation on any component. The characteristics are obtained from a computer program written for either the UNIVAC 1108 or the CDC 6600 computer which requires the inputs of freestream conditions and the airfoil geometry. Similar techniques are needed to handle the complete wing-body-tail combination.

Two recent works should be helpful in predicting the drag coefficients of light aircraft. Roskam in Reference 12 presents two methods for computing drag polars of airplanes at subsonic Mach numbers. The first method models the drag polar by $C_D = C_{D_O} + C_L^2/\pi eAR$ and then sums the zero-lift drag coefficients (usually from wind tunnel data) of each individual component of the aircraft. For a more detailed and accurate drag prediction Roskam suggests a second method. This method employs formulae and charts to estimate the zero-lift drag coefficient of the wing-body combination, the horizontal tail, and the vertical tail as functions of thickness to chord ratio. The drag of the wing-body due to lift is considered to be a function of wing drag due to lift and body drag due to angle of attack; a procedure is also given for estimating incremental drag coefficient due to miscellaneous components such as windshields, nacelles, flaps, etc.

Reference 13 by Wolowicz and Yancey which describes methods for estimating the longitudinal aerodynamic characteristics of light, twin-engine, propeller-driven airplanes, presents a method for estimating the drag coefficient very similar to the second method given by Roskam and discussed above. Also presented are methods for obtaining lift coefficients of the wing, fuselage, horizontal and vertical tails, and interference effects. Most of the methods mentioned above require the use of charts or the manual evaluation of formulas to obtain the lift and drag. A computer program to speed up and mechanize this process would materially simplify point and path performance estimation.

Discussions of point performance are incomplete without some consideration of take-off and landing. NACA TR-450 by Walter S. Diehl (Ref. 14) is concerned with the development of a method suitable for routine take-off calculations which is reasonably simple without neglecting any important variables (See the section on Take-Off and Landing Performance for the general equation). While the method presented in the Technical Report is intended as a practical approximation to a difficult problem, Diehl believed that a more accurate method probably would have no significance in view of the crude state of the lift, drag, and thrust data. Diehl therefore reduced the ground run formula to $S = K_S V_S^2/(T_1/W)$ where V_S is the take-off speed, T_1 is the initial net acceleration force, W is the take-off weight, and K_S is a coefficient depending only on the ratio of initial to final net acceleration force. A relation to estimate the time required to take-off is also given.

A more exact approach to take-off and landing performance is given in Reference 15 prepared by Boeing Aircraft Company. The Boeing report gives a derivation of the basic take-off and landing equation leaving it in integral form. Provided the thrust, lift, drag, and load factor due to rotation during the approach are known, a numerical integration technique can thus be used to evaluate the take-off ground run, time to lift-off, ground distance while climbing to 50 feet, time to climb to 50 feet, ground distance from 50 feet to touchdown, and ground run after touchdown. A detailed discussion of both Reference 14 and Reference 15 is included in the Take-Off and Landing Performance section.

Path Performance

Aircraft can often exceed the equilibrium value of maximum speed, altitude, rate of climb, etc. during periods of accelerated flight. The designer seeking the ultimate in vehicle performance will wish to devise trajectories which maximize particular parameters of interest. One method for doing this, which is growing in popularity with the capability to manipulate and apply it, is the technique of specifying schedules of two control parameters and determining the motion resulting therefrom. The expression "growing in popularity", however, should be used somewhat advisedly. One sees indications of the use of such techniques in the literature and private conversations with industry people also point in the same direction, but specific solutions or calculation procedures are noticeably absent. The results given in Reference 16 represent elementary forms of such procedures.

It is quite probably that several computer programs are currently in use which will compute the trajectory of an aircraft by integrating the first order ordinary differential equations of motion (the integration becomes possibly only when some of the unknown parameters such as power, lift and drag, velocity, etc. are specified as functions of time so as to yield the same number of unknowns as equations). Apparently, the programs are either classified or used only for in-house work by the companies who developed them because none have been described in the open literature. They are thus unavailable to the academic community or the light aircraft industry. For example Reference 17 indicates the existence of a landing analysis digital computer program developed by the Air Force Flight Dynamics Laboratory. This program evolved from a need for comprehensive, quantitative analysis of aircraft take-off and landing characteristics. Similar programs no doubt exist at most of the major aircraft manufacturing companies. Although the light aircraft designer may not need an extremely sophisticated procedure for performance prediction, he should be provided with a procedure which permits him to realize some of the performance improvements resulting from modern technology and which frees him from complete reliance on the basic design charts of the 1930's.

Optimum Performance Paths

Mathematicians have long been concerned with finding the trajectory which optimizes a particular performance criterion. One may cite the classical brachistochrone problem--find the shape of a wire along which a frictionless bead will move under the influence of its own weight from the origin to some other point in minimum time--as a simple example. In attempting to apply variational techniques to the flight of powered aircraft, however, one finds that the additional degrees of freedom present in a realistic mathematical model lead to an almost intractable problem. At first sight, the determination of an optimal range trajectory could appear to be capable of treatment as a classical Mayer problem (see Reference 18 for statement) but mathematical difficulties, apparently encountered by all who have attempted this approach, have proven insurmountable. Those who have employed basically this approach with success have considered more restricted problems such as unpowered flight (Ref. 19).

Within the last 10 years the subject has received intense study because of its applicability to the trajectories of spacecraft. However, one principal feature of many of these analyses--the absence of aerodynamic drag--makes them inapplicable to aircraft use. In general, while the aircraft problems treated by the newer methods of dynamic programming and the Pontryagin Maximum Principle have improved in realism, the techniques are still too complex and too restricted for general computational use. The reader interested in the details of the newer, more successful mathematical methods is directed to References 20, 21, 22, 23, and 24. The last is a particularly good treatment of the subject.

It may be pointed out that while true optimum trajectories can be obtained only with the methods cited, practically speaking there are many near-optimum trajectories which differ little from the optimum in terms of the value of the performance criterion. Some of these near-optimum trajectories can usually be found without excessive difficulty by interative use of the path performance techniques discussed previously.

POINT PERFORMANCE

INTRODUCTION

The process of predicting an aircraft's static or point performance reduces to an investigation of the condition of the flight path assuming that the dependent variables (except for h) of the performance equations do not change with time. If for the general performance equations (Appendix B) the derivatives of the dependent variables are neglected while assuming $\dot{h}$ to be a new dependent variable, a set of non-linear algebraic equations are obtained. The values of the dependent variables required to obtain an optimum flight condition (*i.e.* maximum rate of climb, maximum level flight speed, etc.) can be found by applying the Ordinary Theory of Maxima and Minima.

The work of Oswald (Ref. 3), Rockefeller (Ref. 4), and White and Martin (Ref. 6) as noted earlier represents the state of the art in static performance prediction. Each of these works choose a parabolic relationship between lift and drag in order to obtain a tractable problem while Reference 3 employs in addition some special assumptions regarding the variation of power with velocity for different types of propellers. It is felt, however, that a significant improvement can be made in static performance prediction (1) by using a drag polar which is more general than the conventional parabolic polar and (2) by permitting the user to specify only several points on a curve of maximum power available versus velocity rather than the functional form of the power-velocity relationship. This course has been followed in the present work.

The static performance equations given herein were developed from those in Appendix B by applying the Theory of Maxima and Minima. In addition, a general power versus velocity curve and a drag polar of the form

$$C_D = k_1 + k_2C_L^2 + k_3C_L^{k_4}$$

have been employed. Note that the parabolic polar is a special case of the above polar with k_3 and k_4 equal to zero. The quantities which the analysis considers as known are:

$C_D(C_L) = k_1 + k_2C_L^2 + k_3C_L^{k_4}$ where the user specifies the k's. Alternately the k's may be determined from experimental data using the procedure in Appendix E.

S = wing area on which C_D and C_L are based,

W = airplane weight,

h = altitude at which the optimum characteristics are desired,

P(V) = points on the maximum power available versus velocity curve at specified reference altitude.

Velocity is the unknown and one desires the velocity for which a particular flight characteristic is optimum. An additional restriction imposed on the general equations of motion (Appendix B) for the present analysis is that $\cos \gamma \simeq 1.0$. Should the reader be interested in flight conditions with γ

greater than twelve to fifteen degrees he is referred to the program in Appendix D which integrates the general equations of motion without making the assumption of small values of flight path angle. Once the data defined above is provided the following quantities can be calculated by use of the digital computer program listed in Appendix C:

(1) maximum and minimum level flight speed at an altitude,

(2) speed for maximum climb angle and maximum climb angle at an altitude,

(3) speed for minimum power (maximum endurance) and minimum power at an altitude,

(4) classical speed for maximum range,

(5) service ceiling and the velocity at service ceiling,

(6) absolute ceiling and the velocity at absolute ceiling,

(7) maximum rate of climb schedule from altitude h_1 to altitude h_2,

(8) most economical rate of climb schedule from altitude h_1 to altitude h_2,

(9) maximum rate of climb, power available and power required versus velocity at an altitude.

Note that because of the generalized nature of the drag and power relations employed here, obtaining these optimal quantities usually requires either the solution of a pseudo-polynomial equation having non-integer exponents with velocity as the unknown, or the solution of a pair of the equations with both velocity and altitude as unknowns.

Discussed in detail in succeeding sections are:

(1) the derivation of the optimal static performance equations,

(2) a description of the computerization procedure required to find the solution of these equations,

(3) examples of the static performance calculations for two light aircraft,

(4) a discussion of techniques used to calculate landing and take-off performance.*

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* This discussion is based on two methods contained in the literature and may therefore differ from other methods currently being used. A general computer program to evaluate take-off and landing performance has been omitted because of the difficulty encountered in estimating correct values of C_L, C_D, velocity, and power in these two flight modes.

DERIVATION OF THE POINT PERFORMANCE EQUATIONS

The point or static performance equations are derived by requiring that the acceleration terms be zero. With this restriction the equations developed in Appendix B become

$$x = (V \cos \gamma)\Delta t + x_o \tag{1}$$

$$\dot{h} = V \sin \gamma \tag{2}$$

$$\frac{gP}{WV} - g \frac{S\rho_o}{2} \frac{C_D V^2 \sigma}{W} - g \sin \gamma = 0 \tag{3}$$

$$g \frac{S\rho_o}{2} \frac{C_L V^2 \sigma}{W} - g \cos \gamma = 0 \tag{4}$$

where

$$\sigma = f(h) = (1 - 6.86 \times 10^{-6} h)^{4.26}.$$

(Note that the x equation is given in its integrated form.) In the above set of equations h will be treated as a variable separate from $\dot{h}$; by this device the equation becomes a system of four algebraic equations in nine unknowns (x, V, γ, h, $\dot{h}$, P, W, C_L, C_D). It is customary to specify aircraft weight, power as a function of altitude and velocity, C_L and C_D as a function of angle of attack, and one other parameter in order to make the system solvable.

Now, if one were to seek information on the flight path angle possible at a given altitude he would first express C_D as a general function of C_L,

$$C_D = C_{D_o} + k_1 C_L^2 + k_2 C_L^{k_3}, \tag{5}$$

and then from equation (4) write C_L as

$$C_L = \frac{2W \cos \gamma}{S\rho_o \sigma V^2}, \tag{6}$$

and finally substitute these expressions into equation (3) to yield:

$$\frac{P}{W} - \frac{S\rho_o \sigma V^3}{2W} \left[C_{D_o} + k_1 \left(\frac{2W \cos \gamma}{S\rho_o \sigma V^2}\right)^2 + k_2 \left(\frac{2W \cos \gamma}{S\rho_o \sigma V^2}\right)^{k_3} \right] - V \sin \gamma = 0 . \tag{7}$$

Note that if $k_2 = 0$ and $k_1 = 1/(\pi eAR)$ the drag polar in equation (5) reduces to the familiar parabolic form. Equation (7) expresses γ in terms of V when P, h, and W are given. To find the maximum γ it is necessary to find the value of velocity for which $d\gamma/dV = 0$. The reader will recognize that this is not easily done in closed form. Because of this difficulty, the physics of the situation are invoked to reduce the mathematical complexity. Most general aviation aircraft do not have sufficient power to climb at angles in excess of 12° or 13°; thus, the assumption that $\cos \gamma = 1.0$ is never in error by more than two or three percent. Further, it is difficult to determine C_{D_o} and e with errors of less than four or five percent. It may therefore be argued that no additional error of significance is introduced by taking $\cos \gamma = 1.0$. With this assumption equation (7) becomes

$$\frac{P}{W} - \frac{S\rho_o\sigma V^3}{2W}\left[C_{D_o} + k_1 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^2 + k_2 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^{k_3}\right] - V \sin \gamma = 0. \qquad (8)$$

Finally, through the use of equation (2) $\dot{h}$ can be written as

$$\dot{h} = \frac{P}{W} - \frac{S\rho_o\sigma V^3}{2W}\left[C_{D_o} + k_1 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^2 + k_2 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^{k_3}\right] . \qquad (9)$$

This is the fundamental point performance equation which expresses the rate of climb as a function of speed when P(V,h), W, and h are given.

Maximum and Minimum Level Flight Speed

The maximum and minimum level flight speeds are found by setting $\dot{h} = 0$ in the fundamental equation and solving for V in

$$\frac{P}{W} = \frac{S\rho_o\sigma V^3}{2W}\left[C_{D_o} + k_1 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^2 + k_2 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^{k_3}\right] . \qquad (10)$$

Maximum Rate of Climb

The maximum rate of climb occurs at that speed for which

$$\frac{d\dot{h}}{dV} = \frac{1}{W}\frac{dP}{dV} - \frac{3S\rho_o\sigma V^2}{2W}\left[C_{D_o} + k_1 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^2 + k_2 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^{k_3}\right]$$
$$- \frac{S\rho_o\sigma V^3}{2W}\left[- \frac{4k_1}{V^5}\left(\frac{2W}{S\rho_o\sigma}\right)^2 - \frac{2k_3k_2}{V^{(2k_3 + 1)}}\left(\frac{2W}{S\rho_o\sigma}\right)^{k_3}\right] . \qquad (11)$$

is zero. This speed, substituted into equation (9) gives the maximum rate of climb.

Maximum Climb Angle

An equation for the climb angle can be obtained by noticing that the flight path angle has already been assumed to be small. Hence, $\dot{h} = V \sin \gamma \simeq V \gamma$, or $\gamma = \dot{h}/V$. Division of equation (9) by V therefore yields the desired relation. The speed for the maximum value of γ is then found by setting

$$\frac{d\gamma}{dV} = 0 = \frac{dP}{dV}\frac{1}{WV} - \frac{P}{WV^2} - \frac{S\rho_o\sigma V}{W}\left[C_{D_o} + k_1 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^2 + k_2 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^{k_3}\right]$$

$$- \frac{S\rho_o\sigma V^2}{2W}\left[- \frac{4k_1}{V^5}\left(\frac{2W}{S\rho_o\sigma}\right)^2 - \frac{2k_3k_2}{V^{(2k_3+1)}}\left(\frac{2W}{S\rho_o\sigma}\right)^{k_3}\right] \quad (12)$$

and solving for V. This speed is then substituted into the equation for γ to obtain the maximum climb angle.

Service Ceiling and Absolute Ceiling

The service ceiling is that value of h for which the maximum value of $\dot{h}$ has decreased to 100 feet per minute. The absolute ceiling is that value of h for which $\dot{h} = 0$ at the speed for maximum rate of climb. One must solve two equations simultaneously in order to find the value of V for which σ is a minimum when $\dot{h} = 100$ ft/min or $\dot{h} = 0$. The two equations are repeated below as equations (13) and (14).

$$\dot{h} - \frac{P}{W} + \frac{S\rho_o\sigma V^3}{2W}\left[C_{D_o} + k_1 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^2 + k_2 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^{k_3}\right] = 0 \quad (13)$$

$$\frac{1}{W}\frac{dP}{dV} - \frac{3S\rho_o\sigma V^2}{2W}\left[C_{D_o} + k_1 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^2 + k_2 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^{k_3}\right]$$

$$- \frac{S\rho_o\sigma V^3}{2W}\left[- \frac{4}{V^5} k_1 \left(\frac{2W}{S\rho_o\sigma}\right)^2 - \frac{2k_3k_2}{V^{(2k_3+1)}}\left(\frac{2W}{S\rho_o\sigma}\right)^{k_3}\right] = 0 \quad (14)$$

h is then found from the expression

$$h = \frac{\left(1 - \frac{1}{\sigma^{4.26}}\right)10^6}{6.86} \quad \text{(feet)} .$$

Maximum Range Speed

Fuel consumption in propeller driven aircraft is generally directly proportional to the power required. Thus for maximum range at $\gamma = 0$, one should fly at the speed for which the ratio of power required per unit speed is a minimum

$$\frac{d(P/V)}{dV} = 0 = S\rho_o\sigma V\left[C_{D_o} + k_1 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^2 + k_2 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^{k_3}\right]$$

$$+ \frac{S\rho_o\sigma V^2}{2}\left[- \frac{4k_1}{V^5}\left(\frac{2W}{S\rho_o\sigma}\right)^2 - \frac{2k_3k_2}{V^{(2k_3+1)}}\left(\frac{2W}{S\rho_o\sigma}\right)^{k_3}\right] \quad (15)$$

and solving for V gives the velocity for maximum range. For jet aircraft, the fuel consumption is more appropriately taken to be directly related to the thrust required.

Maximum Endurance

For maximum endurance in propeller-driven aircraft one is interested in flying at the speed for which the power required is a minimum or, the speed which satisfies the equation:

$$\frac{dP}{dV} = 0 = \frac{3}{2} S\rho_o\sigma V^2 \left[C_{D_o} + k_1 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^2 + k_2 \left(\frac{2W}{S\rho_o\sigma V^2}\right)^{k_3}\right]$$

$$+ \frac{S\rho_o\sigma V^3}{2} \left[- \frac{4k_1}{V^5} \left(\frac{2W}{S\rho_o\sigma}\right)^2 - \frac{2k_3k_2}{V^{(2k_3 + 1)}} \left(\frac{2W}{S\rho_o\sigma}\right)^{k_3}\right] . \qquad (16)$$

The minimum power can then be found by substituting this value of velocity into equation (9) with $\dot{h} = 0$.

Minimum Time to Climb

The minimum time to climb is the shortest time required to climb from one altitude to another altitude. It can be expressed in intergral form by

$$T = \int_{h_1}^{h_2} \frac{1}{\dot{h}_{max}} dh \qquad (17)$$

where the velocity for maximum rate of climb is found from equation (11) and the maximum rate of climb is then evaluated using this velocity in equation (9).

Most Economical Climb

The most economical climb is that climb technique which will move an aircraft from h_1 to h_2 while using the least fuel, df. Since df = - dW and $\dot{W}$ = - cP for propeller aircraft, the following procedure may be used to minimize df/dh.

$$\frac{df}{dh} = - \frac{dW}{dh} = - \frac{dW/dt}{dh/dt} = - \frac{\dot{W}}{\dot{h}} = \frac{cP}{\frac{P}{W} - \frac{DV}{W}} = \frac{WcP}{P - DV} .$$

The above expression will have its minimum value when V is a solution to

$$\frac{d(\frac{df}{dh})}{dV} = 0.$$

Thus,

$$\frac{d(\frac{df}{dh})}{dV} = 0 = \frac{d(\frac{WcP}{P - DV})}{dV} = \frac{Wc \frac{dP}{dV}}{P - DV} - \frac{WcP}{(P - DV)^2}\left[\frac{dP}{dV} - V \frac{dD}{dV} - D\right]$$

or,

$$\frac{dP}{dV} DV - PV \frac{dD}{dV} - PD = 0 . \tag{18}$$

Since

$$D = \tfrac{1}{2} \rho_o V^2 S\sigma \left[C_{D_o} + \frac{4W^2 k_1}{(S\rho_o)^2 \sigma^2 V^4} + k_2 \left(\frac{2W}{S\rho_o}\right)^{k_3} \left(\frac{1}{\sigma V^2}\right)^{k_3}\right] \tag{19}$$

then

$$\frac{dD}{dV} = \rho_o S\sigma C_{D_o} V - \frac{4W^2 k_1}{S\rho_o \sigma} \frac{1}{V^3} - \left[(k_3 - 1)(\rho_o S\sigma k_2)\left(\frac{2W}{S\rho_o \sigma}\right)^{k_3}\right] \frac{1}{V^{(2k_3 - 1)}} . \tag{20}$$

Substituting (19) and (20) into (18) yields an equation which can be solved for the velocity for most economical climb. The power used to solve equation (18) is the maximum power available to the aircraft; this is evident from the following sketch of df/dh versus P (here only power greater than DV is considered since this is the minimum power required for climbing flight).

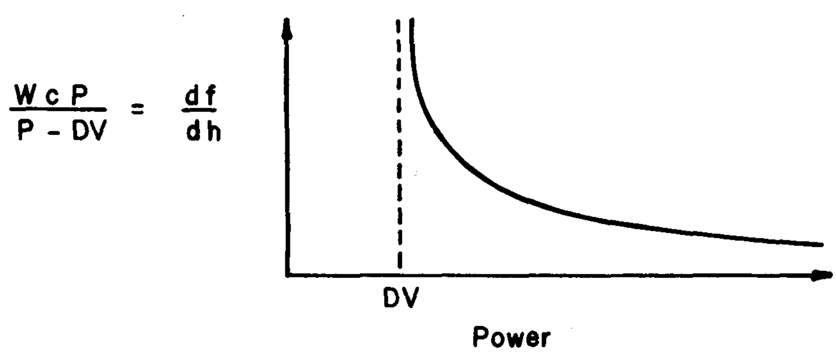

Figure 1. Minimum value of df/dh as a function of power.

df/dh has its minimum value when the power is maximum. Thus, one should fly at the maximum possible power and the velocity given by equation (18) for most economical climb.

COMPUTERIZATION PROCEDURE FOR POINT PERFORMANCE EQUATIONS

A computer program* to evaluate the static performance of an aircraft was written in Fortran IV for use on the IBM 370-165 computer. The procedure employed can best be described by considering the three major portions into which the programming task was divided:

(1) the expression of the maximum power available in a general functional form having a smooth, continuous first derivative given a set of experimental or calculated maximum power-velocity values,

(2) the application of a least-squares-distance curve-fitting technique to fit lift and drag data with a general drag polar of the form

$$C_D = k_1 + k_2C_L^2 + k_3C_L^{k_4} ,$$

(3) the utilization of the method of *regula falsi* (false position) to find the roots of the pseudo-polynomials in velocity derived in the previous section.

These three divisions will now be described in more detail.

Maximum Power Available

For propeller driven aircraft the maximum power available is a function of both velocity and altitude. However, if a maximum power available versus velocity curve is known at some reference altitude the power available at some other altitude may be obtained from the reference curve by means of a multiplicative correction factor (Ref. 3) depending solely on altitude. Denote the power available at velocity V and altitude h by $P_{av}(V,h)$ and the power available at velocity V and the reference altitude h_{ref} by $P_{ref}(V)$. Then, for an unsupercharged engine

$$P_{av}(V,h) = P_{ref}(V)\left(\frac{\sigma - 0.165}{\sigma_{ref} - 0.165}\right)$$

where:

$\sigma = \rho/\rho_o$ = ratio of the density of air at altitude h to the density at sea level,

$\sigma_{ref} = \rho_{ref}/\rho_o$ = ratio of the density of air at reference altitude to the density at sea level.

The variation of the density ratio may be approximated by $\sigma(h) = (1.0 - 6.86 \times 10^{-6}\ h)^{4.26}$ where h is in feet.

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* This program is found in Appendix C along with instruction for its use.

For a supercharged engine the variation of power available with altitude is small up to some fixed altitude which depends on the particular supercharger. Therefore, in this work the altitude variation of power available for a supercharged engine is assumed to be zero, *i.e.* $P_{av}(V,h) = P_{ref}(V)$, and the reference altitude is taken to be sea level.

The general nature of the power available versus velocity curve at the reference altitude considering both fixed pitch and constant speed propeller aircraft makes it very difficult to fix this curve with a constant coefficient polynomial having a smooth, well-behaved first derivative over the entire velocity range. Since the derivative of the power available appears in the static performance equations, a smooth first derivative is a necessity for obtaining valid solutions to these equations. The spline technique has been shown to give the best mathematical fit for a set of data points, and the cubic spline is the simplest fit which provides a well-behaved first derivative. For this reason the maximum power available data points were fitted using the cubic spline given in Reference 25 with modified end conditions.

The power available at some velocity V at the reference altitude was denoted above by $P_{ref}(V)$. Let the data points from the reference power available versus velocity curve be denoted by $(P_{ref})_i$, V_i, $i = 1,2,\ldots,N$, where N is the number of data points. Then the cubic spline fit is of the form

$$P_{ref}(V) = C_{1_j} V^3 + C_{2_j} V^2 + C_{3_j} V + C_{4_j}$$

where:

$$V_j \le V \le V_{j+1}, \quad \text{and } j = 1,2,\ldots,N-1$$

Note that for each of the N-1 intervals the cubic's coefficients depend on the interval (V_j, V_{j+1}) in which the velocity V lies, but they are constant in a given interval. It is this variation of the coefficients that gives the spline fit its remarkable curve fitting properties.

To develop a suitable maximum-power-available-at-any-altitude relationship for use in the static performance calculations, the computer program* (Appendix C) requires that the user supply only a set of power available versus velocity data points at a reference altitude, the reference altitude, and a control parameter denoting whether or not the engine is supercharged. A technique for estimating these data points is given in Appendix F.

Lift-Drag Curve Fitting Technique

Most previous estimation techniques for static performance have relied on a conventional parabolic drag polar. Because some performance parameters

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* The program in Appendix D also used this procedure to obtain maximum power available.

are evaluated at relatively high angles of attack where the drag is frequently greater than predicted by a parabolic fit to high speed data, the use of the parabolic polar for such computations leads to erroneous results. Accordingly, the static performance analysis in the preceeding section permits the use of a general drag polar of the form

$$C_D = k_1 + k_2 C_L^2 + k_3 C_L^{k_4} .$$

Note that this general drag polar includes the parabolic polar as a special case, *i.e.* $k_3 = k_4 = 0$.

If lift and drag data, preferably up to $C_{L_{max}}$, are available, the coefficients of the general drag polar can be obtained with the program given in Appendix E. This program, a modification of the one given in Reference 26, uses a least-squares-distance technique to fit the data, *i.e.* it minimizes the sum of the squares of the perpendicular distances from the data point to the fitted curve. This type of least squares technique is desirable because the drag coefficient versus lift coefficient curve has regions of both small and large slopes.

The program gives the user the option of the following four particular forms of this general drag polar:

(1) $C_D = k_1 + k_2 C_L^2 + k_3 C_L^{k_4}$

(2) $C_D = C_{D_o} + k_2 C_L^2 + k_3 C_L^{k_4}$

(3) $C_D = k_1 + k_3 C_L^{k_4}$ $\quad (k_2 = 0)$

(4) $C_D = C_{D_o} + k_3 C_L^{k_4}$ $\quad (k_2 = 0)$

In cases (1) and (3) all coefficients are varied in the fitting process, and k_1 may not be the actual zero-lift drag coefficient. In cases (2) and (4) the user specifies C_{D_O}, the zero-lift drag coefficient, and it is not varied in the fitting process.*

Solution of the Pseudo-Polynomials

The determination of each performance parameter requires the solution of a pseudo-polynomial** in velocity, except the cases of service ceiling and absolute ceiling where the simultaneous solution of two coupled

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* Cases (3) and (4) are the drag polar forms used in the program which integrates the equations of motion (Appendix D).

** The word pseudo-polynomial refers to a polynomial which has the unknown raised to powers which are not necessarily integers.

pseudo-polynomials, one in velocity with its coefficients depending on altitude and one in altitude with its coefficients depending on velocity, are required.

For those cases where the solution of a single pseudo-polynomial is required, the method of *regula falsi* (false position) is used (Ref. 27). The velocity range of the power available versus velocity curve is searched by increments until a sign change of the pseudo-polynomial occurs. The false position method is then used to obtain the zero of the pseudo-polynomial to within a specified tolerance. Note that since the spline curve fit (see above section) of the power available versus velocity gives a cubic polynomial with different coefficients from each velocity interval, the coefficients of the various pseudo-polynomials will vary as the entire velocity range is searched.

In the case of service or absolute ceilings the method of false position is used in conjunction with an iteration between the velocity polynomial and the altitude polynomial. This iterative procedure is as follows:

(1) A service or absolute ceiling is assumed, and the velocity pseudo-polynomial is solved by *regula falsi*.

(2) With this velocity root the altitude pseudo-polynomial is solved by *regula falsi* for a new value of the service or absolute ceiling.

(3) With this new altitude step (1) is repeated. Iteration continues between step (1) and step (2) until convergence on both velocity and altitude is achieved to within a specified tolerance.

EXAMPLES OF POINT PERFORMANCE CALCULATIONS

In order to evaluate the static performance program, two single engine light aircraft were investigated; both the description and the results of these two test cases are given below.

The performance of the Cessna 182 (Figure 2) was evaluated using a conventional parabolic drag polar and a polar obtained by curve fitting lift and drag coefficients obtained from Cessna Aircraft through personal communication. The reference wing area for the lift and drag coefficients was 174 square feet while the basic weight was 2650 pounds. The maximum power available curve was obtained using the procedure outlined in Appendix F. The engine, a Continental Model 0-470-R, had a power rating of 230 BHP at 2600 RPM. The maximum engine speed for continuous operation was assumed to be 2400 RPM. The propeller had a R.A.F. 6 section with a diameter of seven feet.

Tables 1 and 2 may be used to compare the performance of the Cessna 182 with the two different drag polars; these tables present the performance characteristics for the parabolic and the fitted drag polars respectively. The major difference is seen when comparing the minimum level flight speeds. The analysis with a parabolic polar gives a much lower value of minimum level flight speed than does the fitted polar; this is caused by the error encountered when using a parabolic polar at large lift coefficients (high angles of attack).

The program may be easily adapted to find variations of static performance parameters for different values of aircraft weight and altitude. The power available and required curves at sea level, 8000 feet, and 16000 feet are presented in Figure 3 for the Cessna 182 with the fitted drag polar. The variations in maximum and minimum level flight speeds and maximum rate of climb with weight and altitude are given in Figure 4. Figure 5 has also been included to indicate how the changes in weight affect the service and absolute ceilings.

Because of the availability of lift and drag data (Ref. 28) the performance of the Navion was evaluated using the static performance program. Several points from the wind tunnel data were used to find a general drag polar*, while a 285 BHP engine rate at 2900 RPM was used at the power plant. The results of this analysis are given in Table 3.

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* The sample output given in Appendix E contains the actual data for the Navion aircraft.

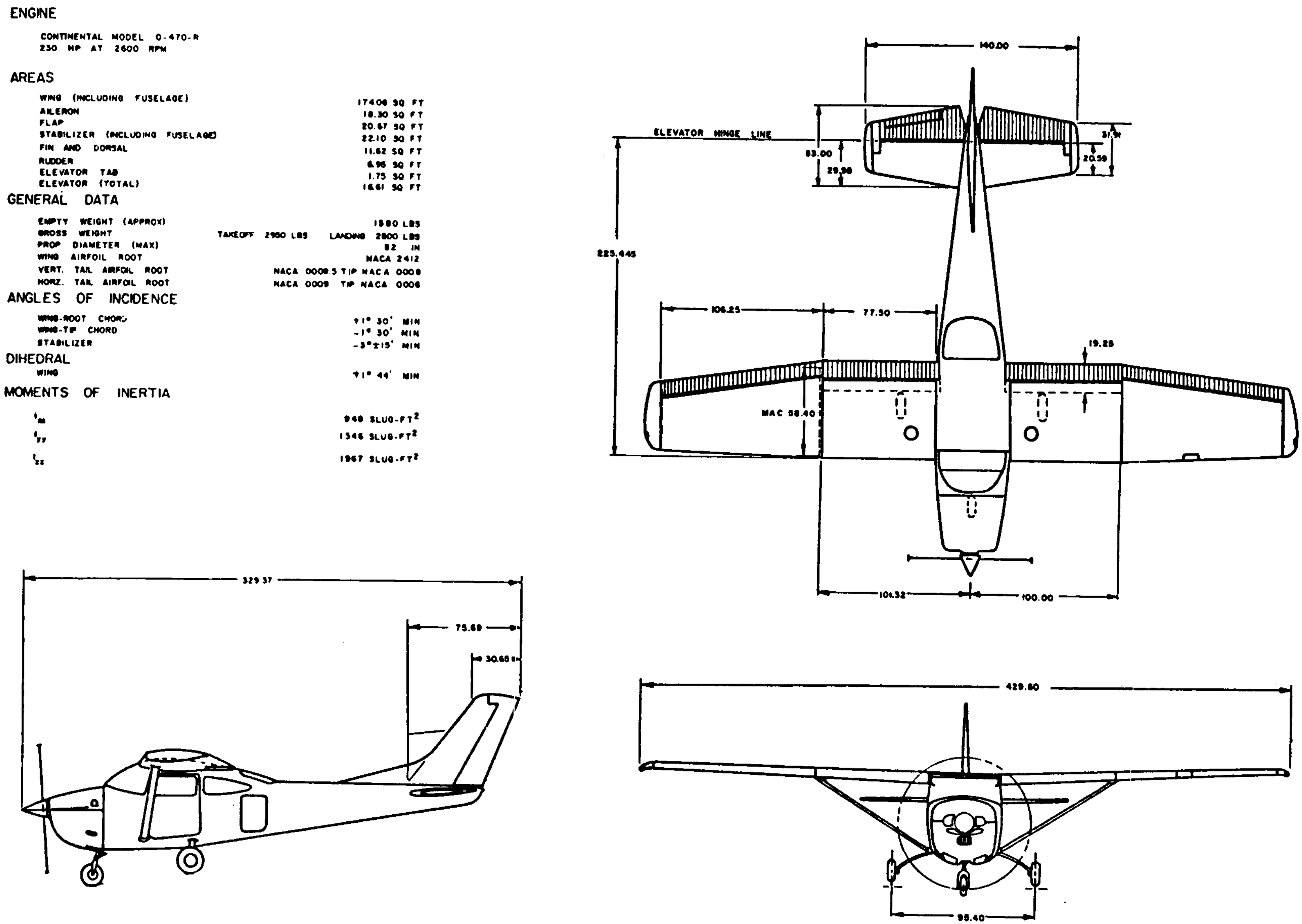


Figure 2. Three view of the Cessna 182.

POWER AVAILABLE VS. VELOCITY
REFERENCE ALTITUDE = 0.0 FEET

PA(FT-LBS/SEC)	V(FT/SEC)
0.0	0.0
0.291500 05	0.273300 02
0.524700 05	0.546700 02
0.699600 05	0.820000 02
0.816200 05	0.109330 03
0.874500 05	0.136670 03
0.909480 05	0.164000 03
0.944460 05	0.191330 03
0.958650 05	0.218670 03
0.967780 05	0.246000 03
0.973610 05	0.273330 03
0.979440 05	0.300670 03
0.994700 05	0.328000 03
0.994700 05	0.355330 03
0.994700 05	0.382660 03

AIRCRAFT CHARACTERISTICS

CD = 0.269000-01 + 0.440240-01*CL**2 + 0.0 *CL** 0.0
WING AREA = 0.174C00 03 SQ.FT WEIGHT = 0.265000 04 LBS

STATIC PERFORMANCE AT AN ALTITUDE = 0.0 FT
WITH MINIMUM TIME AND MOST ECONOMICAL CLIMB SCHEDULES TO A FINAL ALTITUDE = 0.10000D 05 FT
++

MINIMUM LEVEL FLIGHT SPEED = 0.383470 02 FT/SEC
LIFT COEFFICIENT = 0.870360 01 DRAG COEFFICIENT = 0.336180 01

MAXIMUM LEVEL FLIGHT SPEED = 0.253850 03 FT/SEC
LIFT COEFFICIENT = 0.198610 00 DRAG COEFFICIENT = 0.286370-01

MAXIMUM CLIMB ANGLE = 0.128580 02 DEG
VELOCITY FOR MAXIMUM CLIMB ANGLE = 0.853090 02 FT/SEC
LIFT COEFFICIENT = C.175860 01 DRAG COEFFICIENT = 0.16305C 00

VELOCITY FOR MAXIMUM ENDURANCE = 0.972250 02 FT/SEC
POWER FOR MAXIMUM ENDURANCE = 0.204760 05 FT-LBS/SEC
LIFT COEFFICIENT = 0.135390 01 DRAG COEFFICIENT = 0.107600 00

VELOCITY FOR CLASSICAL MAXIMUM RANGE = 0.127960 03 FT/SEC
LIFT COEFFICIENT = 0.781680 00 DRAG COEFFICIENT = 0.538000-01

SERVICE CEILING = 0.228310 05 FT
VELOCITY AT SERVICE CEILING = 0.150450 03 FT/SEC
LIFT COEFFICIENT = 0.116810 01 DRAG COEFFICIENT = 0.869700-01

ABSOLUTE CEILING = 0.248760 05 FT
VELOCITY AT ABSOLUTE CEILING = 0.154700 03 FT/SEC
LIFT COEFFICIENT = 0.118670 01 DRAG COEFFICIENT = 0.888960-01

MAXIMUM RATE OF CLIMB SCHEDULE FROM 0.0 FT TO 0.100000 05 FT

H(FT)	R/C(FT/SEC)	V(FT/SEC)	P(FT-LBS/SEC)	CL	CD	T(SEC)
0.0	0.236640 02	0.127650 03	0.859940 05	0.785380 00	0.540550-01	0.0
0.500000 03	0.230980 02	0.127860 03	0.845340 05	0.794390 00	0.546820-01	0.213880 02
0.100000 04	0.225370 02	0.128070 03	0.830900 05	0.803460 00	0.553190-01	0.433040 02
0.150000 04	0.219800 02	0.128300 03	0.816630 05	0.812570 00	0.559680-01	0.657710 02
0.200000 04	0.214270 02	0.128520 03	0.802520 05	0.821740 00	0.566270-01	0.888130 02
0.250000 04	0.208790 02	0.128760 03	0.788580 05	0.830950 00	0.572970-01	0.112450 03
0.300000 04	0.203350 02	0.129010 03	0.774790 05	0.840200 00	0.579780-01	0.136720 03
0.350000 04	0.197950 02	0.129260 03	0.761160 05	0.849490 00	0.586690-01	0.161650 03
0.400000 04	0.192600 02	0.129530 03	0.747690 05	0.858810 00	0.593700-01	0.187250 03
0.450000 04	0.187290 02	0.129800 03	0.734370 05	0.868160 00	0.600810-01	0.213580 03
0.500000 04	0.182020 02	0.130090 03	0.721200 05	0.877530 00	0.608010-01	0.240670 03
0.550000 04	0.176790 02	0.130380 03	0.708190 05	0.886920 00	0.615300-01	0.268540 03
0.600000 04	0.171600 02	0.130680 03	0.695330 05	0.896320 00	0.622680-01	0.297250 03
0.650000 04	0.166450 02	0.131000 03	0.682620 05	0.905730 00	0.630150-01	0.326840 03
0.700000 04	0.161340 02	0.131330 03	0.670060 05	0.915140 00	0.637690-01	0.357350 03
0.750000 04	0.156270 02	0.131670 03	0.657640 05	0.924540 00	0.645310-01	0.388850 03
0.800000 04	0.151230 02	0.132020 03	0.645370 05	0.933930 00	0.652990-01	0.421380 03
0.850000 04	0.146230 02	0.132380 03	0.633240 05	0.943300 00	0.660730-01	0.455000 03
0.900000 04	0.141270 02	0.132760 03	0.621260 05	0.952650 00	0.668530-01	0.489800 03
0.950000 04	0.136350 02	0.133150 03	0.609420 05	0.961960 00	0.676380-01	0.525830 03
0.100000 05	0.131460 02	0.133550 03	0.597710 05	0.971220 00	0.684270-01	0.563180 03

Table 1. Performance of Cessna 182 with parabolic drag polar.

POWER AVAILABLE VS. VELOCITY
REFERENCE ALTITUDE = 0.0 FEET

PA(FT-LBS/SEC)	V(FT/SEC)
0.0	0.0
0.29150D 05	0.27330D 02
0.52470D 05	0.54670D 02
0.69960D 05	0.82000D 02
0.81620D 05	0.10933D 03
0.87450D 05	0.13667D 03
0.90948D 05	0.16400D 03
0.94446D 05	0.19133D 03
0.95865D 05	0.21867D 03
0.96778D 05	0.24600D 03
0.97361D 05	0.27333D 03
0.97944D 05	0.30067D 03
0.99470D 05	0.32800D 03
0.99470D 05	0.35533D 03
0.99470D 05	0.38266D 03

AIRCRAFT CHARACTERISTICS

CD = 0.26880D-01 + 0.54242D-01*CL**2 + 0.17751D-01*CL** 0.65000D 01
WING AREA = 0.17400D 03 SQ.FT WEIGHT = 0.26500D 04 LBS

STATIC PERFORMANCE AT AN ALTITUDE = 0.0 FT
WITH MINIMUM TIME AND MOST ECONOMICAL CLIMB SCHEDULES TO A FINAL ALTITUDE = 0.10000D 05 FT
++

MINIMUM LEVEL FLIGHT SPEED = 0.90465D 02 FT/SEC
LIFT COEFFICIENT = 0.15638D 01 DRAG COEFFICIENT = 0.48418D 00

MAXIMUM LEVEL FLIGHT SPEED = 0.25257D 03 FT/SEC
LIFT COEFFICIENT = 0.20062D 00 DRAG COEFFICIENT = 0.29063D-01

MAXIMUM CLIMB ANGLE = 0.10220D 02 DEG
VELOCITY FOR MAXIMUM CLIMB ANGLE = 0.11710D 03 FT/SEC
LIFT COEFFICIENT = 0.93338D 00 DRAG COEFFICIENT = 0.85475D-01

VELOCITY FOR MAXIMUM ENDURANCE = 0.12571D 03 FT/SEC
POWER FOR MAXIMUM ENDURANCE = 0.27545D 05 FT-LBS/SEC
LIFT COEFFICIENT = 0.80985D 00 DRAG COEFFICIENT = 0.66961D-01

VELOCITY FOR CLASSICAL MAXIMUM RANGE = 0.14205D 03 FT/SEC
LIFT COEFFICIENT = 0.63423D 00 DRAG COEFFICIENT = 0.49619D-01

SERVICE CEILING = 0.19442D 05 FT
VELOCITY AT SERVICE CEILING = 0.17554D 03 FT/SEC
LIFT COEFFICIENT = 0.76424D 00 DRAG COEFFICIENT = 0.61653D-01

ABSOLUTE CEILING = 0.21236D 05 FT
VELOCITY AT ABSOLUTE CEILING = 0.18023D 03 FT/SEC
LIFT COEFFICIENT = 0.77056D 00 DRAG COEFFICIENT = 0.62348D-01

MAXIMUM RATE OF CLIMB SCHEDULE FROM 0.0 FT TO 0.10000D 05 FT

H(FT)	R/C(FT/SEC)	V(FT/SEC)	P(FT-LBS/SEC)	CL	CD	T(SEC)
0.0	0.22257D 02	0.13601D 03	0.87354D 05	0.69183D 00	0.54460D-01	0.0
0.50000D 03	0.21659D 02	0.13662D 03	0.85921D 05	0.69581D 00	0.54821D-01	0.22775D 02
0.10000D 04	0.21065D 02	0.13725D 03	0.84504D 05	0.69966D 00	0.55174D-01	0.46185D 02
0.15000D 04	0.20475D 02	0.13789D 03	0.83102D 05	0.70339D 00	0.55519D-01	0.70263D 02
0.20000D 04	0.19889D 02	0.13856D 03	0.81716D 05	0.70700D 00	0.55856D-01	0.95042D 02
0.25000D 04	0.19307D 02	0.13925D 03	0.80345D 05	0.71049D 00	0.56185D-01	0.12056D 03
0.30000D 04	0.18728D 02	0.13996D 03	0.78989D 05	0.71384D 00	0.56504D-01	0.14686D 03
0.35000D 04	0.18154D 02	0.14069D 03	0.77647D 05	0.71708D 00	0.56814D-01	0.17398D 03
0.40000D 04	0.17583D 02	0.14145D 03	0.76320D 05	0.72018D 00	0.57115D-01	0.20197D 03
0.45000D 04	0.17015D 02	0.14222D 03	0.75008D 05	0.72316D 00	0.57405D-01	0.23088D 03
0.50000D 04	0.16451D 02	0.14302D 03	0.73709D 05	0.72600D 00	0.57684D-01	0.26077D 03
0.55000D 04	0.15891D 02	0.14384D 03	0.72425D 05	0.72872D 00	0.57953D-01	0.29170D 03
0.60000D 04	0.15335D 02	0.14468D 03	0.71155D 05	0.73131D 00	0.58211D-01	0.32373D 03
0.65000D 04	0.14782D 02	0.14554D 03	0.69899D 05	0.73376D 00	0.58457D-01	0.35695D 03
0.70000D 04	0.14233D 02	0.14643D 03	0.68657D 05	0.73608D 00	0.58691D-01	0.39142D 03
0.75000D 04	0.13687D 02	0.14734D 03	0.67428D 05	0.73828D 00	0.58914D-01	0.42726D 03
0.80000D 04	0.13144D 02	0.14828D 03	0.66213D 05	0.74034D 00	0.59124D-01	0.46454D 03
0.85000D 04	0.12606D 02	0.14924D 03	0.65011D 05	0.74227D 00	0.59322D-01	0.50339D 03
0.90000D 04	0.12070D 02	0.15022D 03	0.63823D 05	0.74407D 00	0.59508D-01	0.54394D 03
0.95000D 04	0.11539D 02	0.15122D 03	0.62649D 05	0.74573D 00	0.59681D-01	0.58632D 03
0.10000D 05	0.11010D 02	0.15226D 03	0.61488D 05	0.74727D 00	0.59841D-01	0.63069D 03

Table 2. Performance of the Cessna 182 with the the general drag polar.

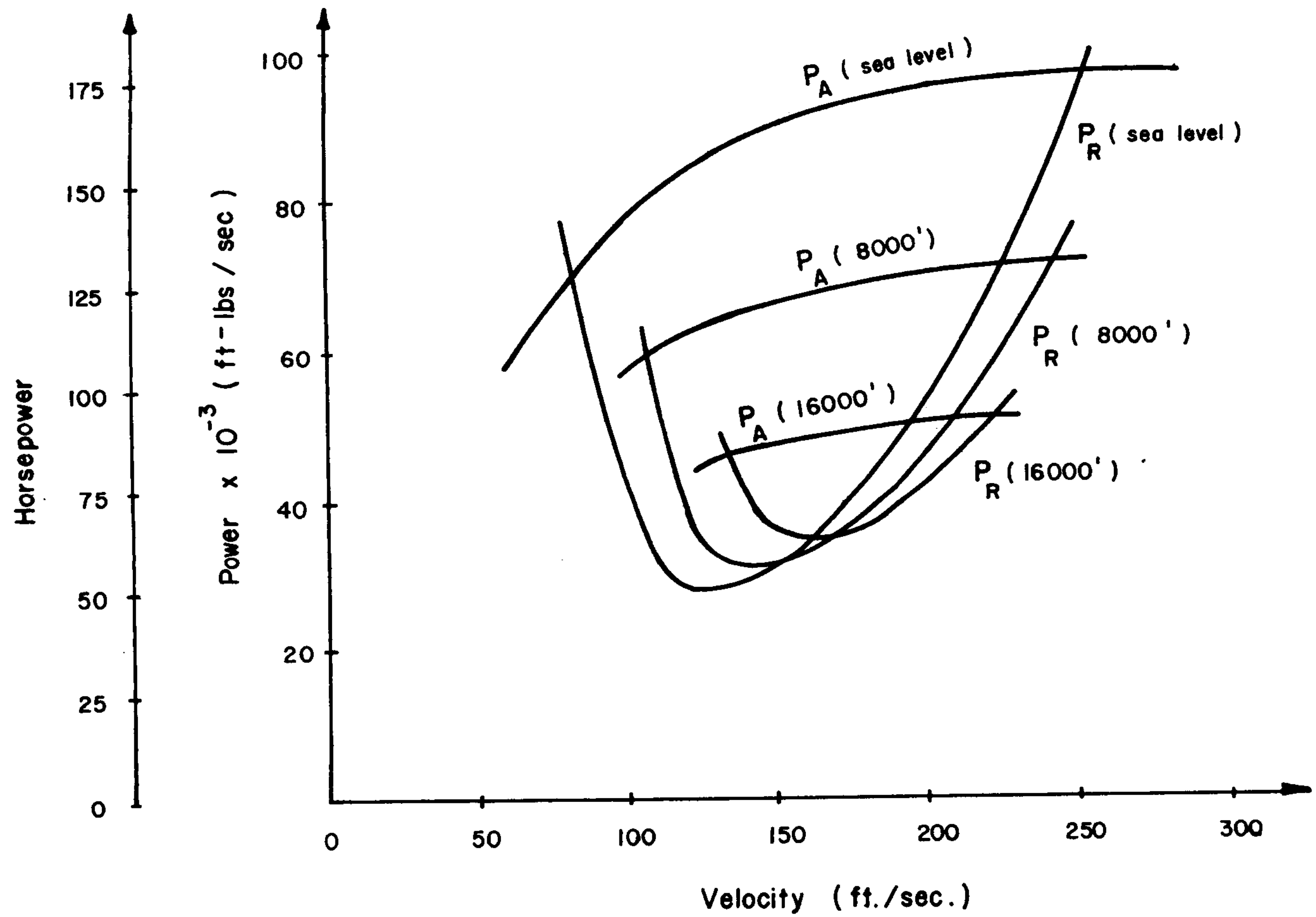
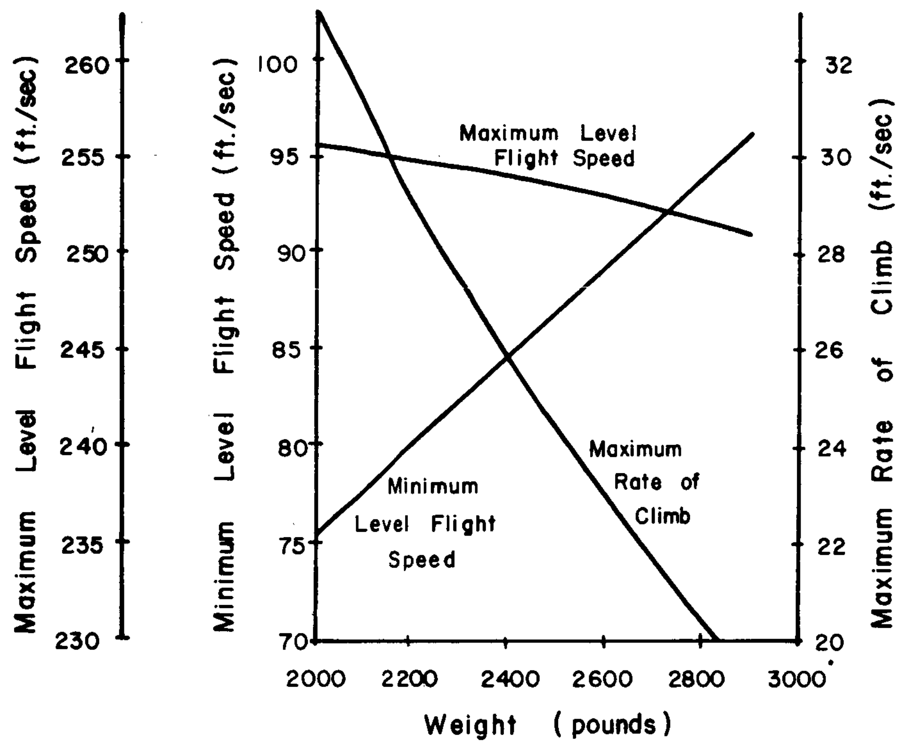


Figure 3. Power required and power available versus velocity.



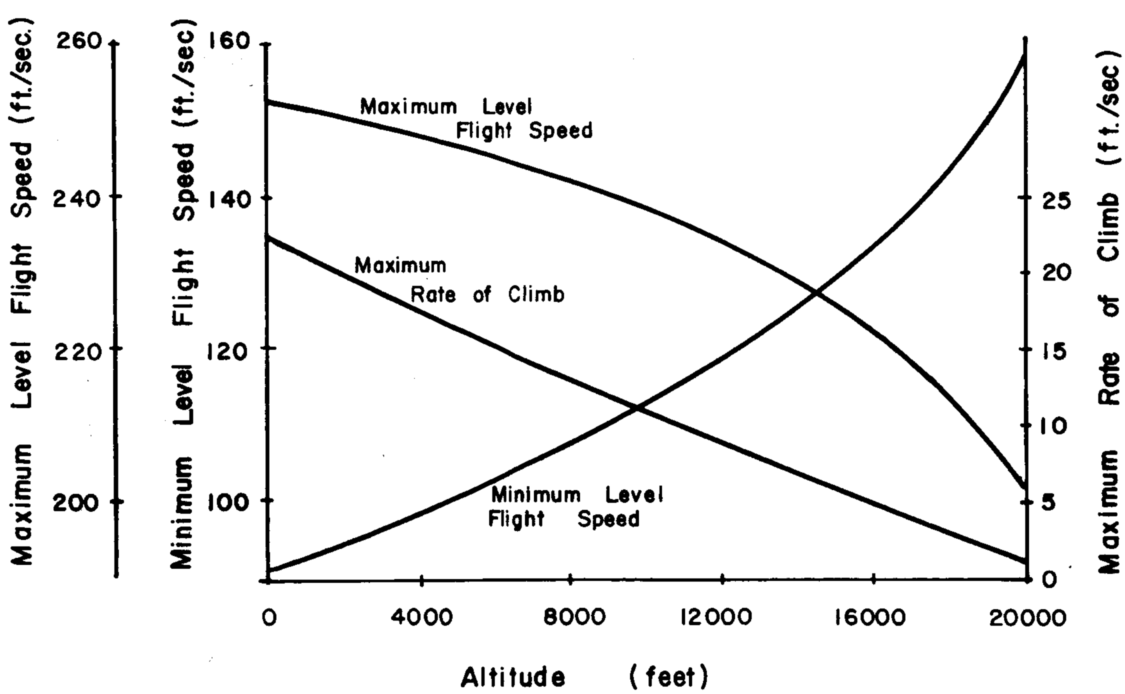


Figure 4. Effects of weight and altitude on maximum and minimum level flight speeds and maximum rate of climb.

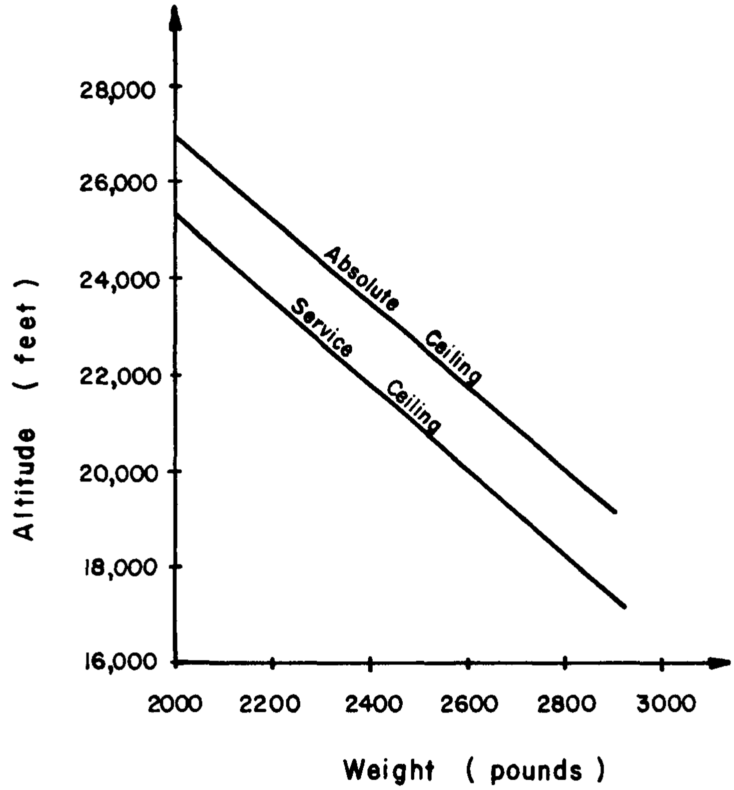


Figure 5. Service and absolute ceilings for various weights.

POWER AVAILABLE VS. VELOCITY
REFERENCE ALTITUDE = 0.0 FEET

PA(FT-LBS/SEC)	V(FT/SEC)
0.0	0.0
0.36300D 05	0.28150D 02
0.65340D 05	0.56300D 02
0.87120D 05	0.84450D 02
0.10164D 06	0.11260D 03
0.10890D 06	0.14075D 03
0.11326D 06	0.16890D 03
0.11761D 06	0.19705D 03
0.11979D 06	0.22520D 03
0.12052D 06	0.25335D 03
0.12124D 06	0.28150D 03
0.12197D 06	0.30965D 03
0.12342D 06	0.33780D 03
0.12342D 06	0.36595D 03
0.12342D 06	0.39410D 03

AIRCRAFT CHARACTERISTICS

CD = 0.47000D-01 + 0.10153D-01*CL**2 + 0.43478D-01*CL** 0.54116D 01
WING AREA = 0.18000D 03 SQ.FT WEIGHT = 0.27500D 04 LBS

STATIC PERFORMANCE AT AN ALTITUDE = 0.0 FT
WITH MINIMUM TIME AND MOST ECONOMICAL CLIMB SCHEDULES TO A FINAL ALTITUDE = 0.10000D 05 FT
++

MINIMUM LEVEL FLIGHT SPEED = 0.90382D 02 FT/SEC
LIFT COEFFICIENT = C.15716D 01 DRAG COEFFICIENT = 0.57419D 00

MAXIMUM LEVEL FLIGHT SPEED = 0.22731D 03 FT/SEC
LIFT COEFFICIENT = C.24848D 00 DRAG COEFFICIENT = 0.47650D-01

MAXIMUM CLIMB ANGLE = 0.13081D 02 DEG
VELOCITY FOR MAXIMUM CLIMB ANGLE = 0.11727D 03 FT/SEC
LIFT COEFFICIENT = C.93349D 00 DRAG COEFFICIENT = 0.85806D-01

VELOCITY FOR MAXIMUN ENDURANCE = 0.12351D 03 FT/SEC
POWER FOR MAXIMUM ENDURANCE = 0.28772D 05 FT-LBS/SEC
LIFT COEFFICIENT = C.84164D 00 DRAG COEFFICIENT = 0.71296D-01

VELOCITY FOR CLASSICAL MAXIMUM RANGE = 0.13060D 03 FT/SEC
LIFT COEFFICIENT = C.75276D 00 DRAG COEFFICIENT = 0.62103D-01

SERVICE CEILING = 0.22106D 05 FT
VELOCITY AT SERVICE CEILING = 0.17782D 03 FT/SEC
LIFT COEFFICIENT = 0.81810D 00 DRAG COEFFICIENT = 0.68464C-01

ABSOLUTE CEILING = 0.23725D 05 FT
VELOCITY AT ABSOLUTE CEILING = 0.18262D 03 FT/SEC
LIFT COEFFICIENT = C.82C44D 00 DRAG COEFFICIENT = 0.68732D-01

MAXIMUM RATE OF CLIMB SCHEDULE FROM 0.0 FT TO 0.10000D 05 FT

H(FT)	R/C(FT/SEC)	V(FT/SEC)	P(FT-LBS/SEC)	CL	CD	T(SEC)
0.0	0.28095D 02	0.13050D 03	0.10687D 06	0.75389D 00	0.62196D-01	0.0
0.50000D 03	0.27416D 02	0.13116D 03	0.10515D 06	0.75727D 00	0.62479D-01	0.18017D 02
0.10000D 04	0.26741D 02	0.13184D 03	0.10346D 06	0.76055D 00	0.62758D-01	0.36485D 02
0.15000D 04	0.26070D 02	0.13254D 03	0.10177D 06	0.76371D 00	0.63031D-01	0.55424D 02
0.20000D 04	0.25402D 02	0.13326D 03	0.10011D 06	0.76676D 00	0.63299D-01	0.74855D 02
0.25000D 04	0.24739D 02	0.13400D 03	0.98462D 05	0.76970D 00	0.63561D-01	0.94802D 02
0.30000D 04	0.24079D 02	0.13475D 03	0.96829D 05	0.77253D 00	0.63817D-01	0.11529D 03
0.35000D 04	0.23423D 02	0.13552D 03	0.95213D 05	0.77525D 00	0.64066D-01	0.13635D 03
0.40000D 04	0.22771D 02	0.13632D 03	0.93612D 05	0.77786D 00	0.64308D-01	0.15800D 03
0.45000D 04	0.22123D 02	0.13712D 03	0.92027D 05	0.78036D 00	0.64544D-01	0.18028D 03
0.50000D 04	0.21478D 02	0.13795D 03	0.90457D 05	0.78275D 00	0.64771D-01	0.20322D 03
0.55000D 04	0.20837D 02	0.13880D 03	0.88903D 05	0.78503D 00	0.64991D-01	0.22685D 03
0.60000D 04	0.20200D 02	0.13967D 03	0.87364D 05	0.78721D 00	0.65203D-01	0.25123D 03
0.65000D 04	0.19567D 02	0.14055D 03	0.85840D 05	0.78928D 00	0.65407D-01	0.27638D 03
0.70000D 04	0.18938D 02	0.14146D 03	0.84331D 05	0.79125D 00	0.65602D-01	0.30236D 03
0.75000D 04	0.18312D 02	0.14238D 03	0.82836D 05	0.79313D 00	0.65791D-01	0.32921D 03
0.80000D 04	0.17690D 02	0.14332D 03	0.81356D 05	0.79493D 00	0.65973D-01	0.35700D 03
0.85000D 04	0.17071D 02	0.14428D 03	0.79891D 05	0.79665D 00	0.66149D-01	0.38577D 03
0.90000D 04	0.16457D 02	0.14525D 03	0.78441D 05	0.79828D 00	0.66317D-01	0.41561D 03
0.95000D 04	0.15846D 02	0.14625D 03	0.77004D 05	0.79983D 00	0.66478D-01	0.44658D 03
0.10000D 05	0.15239D 02	0.14726D 03	0.75583D 05	0.80131D 00	0.66631D-01	0.47876D 03

Table 3. Performance of the Navion for a general drag polar.

The static performance previously discussed has been concerned primarily with the cruising and climbing aircraft. However, the complete performance analysis must also include a discussion of take-off and landing. Take-off performance analysis can usually be divided into two parts, ground run and climb over a 50 foot obstacle. Analogously, landing consists of the approach (from a 50 foot altitude to touch-down) and the ground run. A brief analysis of both take-off and landing performance taken largely from Reference 15 is presented below.

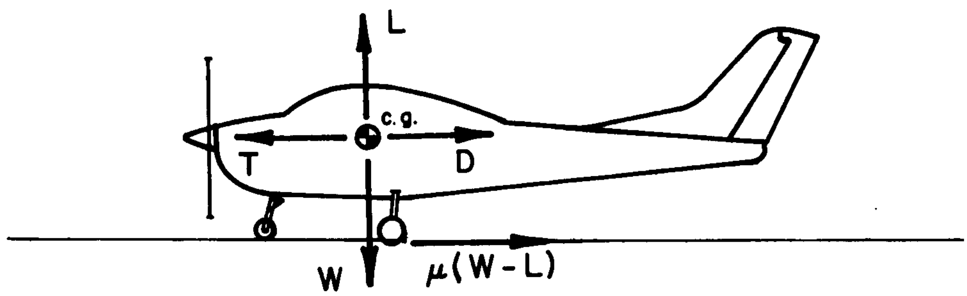


Figure 6. Forces acting on an aircraft during landing and take-off.

Take-Off Ground Run

The distance traversed by an airplane on level ground in accelerating from one speed to another can be expressed by

$$ds = \frac{VdV}{\bar{a}} ,$$

where

$$V = \frac{ds}{dt} \text{ and } \bar{a} = \frac{dV}{dt} .$$

The ground run can thus be found by integrating the above expression to yield

$$S_G = s_y - s_x = \int_{V_x}^{V_y} \frac{VdV}{\bar{a}} . \tag{21}$$

If the airplane is starting from rest $V_x = 0$ and $V_y = V_{LOF}$ = velocity at lift-off. The effects of wind on take-off can be accounted for by letting $V_x = V_w$ = velocity of the headwind and by replacing V by $(V - V_w)$ since $\bar{a}$ is a function of ground velocity $(V - V_w)$.

The frictional force of the wheels on the runway is a force which acts in the same direction as the drag force. This force is proportional to the weight less the lift and is given by

$$F_f = (W - L)\,\mu$$

where μ is the coefficient of friction. The value of μ depends upon the type of runway surface used. Typical values of μ are given below (Ref. 29):

SURFACE	μ
Concrete, asphalt, or wood	0.02
Hard turf	0.04
Average field - short grass	0.05
Average field - long grass	0.10
Soft ground	0.10 → 0.30

Table 4. Typical values of μ for various runway surfaces.

In addition to the frictional force opposing the thrust there is a drag force so that the acceleration during the ground run is given by

$$\bar{a} = \frac{g[T - D - \mu(W - L)]}{W} \quad . \tag{22}$$

Expressing drag and lift in coefficient form the distance equation (21) can be expressed as

$$S_G = \int_{V_w}^{V_{LOF}} \frac{W}{g} \frac{(V - V_w)\, dV}{[T - \mu W - (C_D - \mu C_L)\, \frac{1}{2}\, \rho_o \sigma V^2]} , \tag{23}$$

where S_G is the ground distance required for take-off. For take-off C_D, C_L, and W can be assumed to be independent of velocity and the thrust, T, can be found from P/V where P will be the maximum power. Since $dt = dV/\bar{a}$, the time to lift-off can be expressed as:

$$t_G = \int_{V_w}^{V_{LOF}} \frac{W}{g} \frac{dV}{[T - \mu W - (C_D - \mu C_L)\, \frac{1}{2}\, \rho_o \sigma V^2 S]} \quad . \tag{24}$$

Since a numerical integration technique is required to evalaute the integrals given in Equations (23) and (24), a comparatively simple method for predicting the take-off ground run and the time to lift-off is also offered. This method (Ref. 14), presented in 1933, was intended as a practical approximation to a difficult problem. The steps proceed as follows:

(1) Evaluate $$\frac{T_1}{W} = \frac{(K_{T_o})(BHP)}{(W)(N)(D)} - \mu ,$$

and $$\frac{T_F}{W} = \frac{(550)(thpm)}{W\ V_s} \left(\frac{thp}{thp_m}\right) - \frac{D}{L}$$

where:

KT_o = static thrust coefficient (Figure 7 or 8)
BHP = engine brake horsepower
N = engine speed in revolutions per minute
D = propeller diameter in feet
W = take-off weight in pounds
μ = coefficient of wheel friction (Table 4)
thp_m = maximum thrust horsepower

$\left(\frac{thp}{thp_m}\right)$ = ratio of thrust horsepower at speed V to thrust horsepower at maximum speed (Figure 9)

$\frac{D}{L}$ = the reciprocal of the maximum value of L/D which is either known or estimated (Figure 10)

V_s = take-off velocity.

(2) Evaluate $$S_o = \frac{K_s\ V_s^2}{\left(\frac{T_1}{W}\right)} \quad \text{and} \quad t_o = \frac{K_t\ V_s}{\left(\frac{T_1}{W}\right)}$$

where:

S_o = take-off ground run
t_o = time required to lift-off
K_s = take-off coefficient in Figure 11 as a function of $(T_F/W)/(T_1/W)$
K_t = time coefficient in Figure 12 as a function of $(T_F/W)/(T_1/W)$.

(3) Corrections for take-off headwind and take-off weight changes can be obtained from Figures 13 and 14 by using the two relations given below:

$$S_w = S_o \left(\frac{S_w}{S_o}\right) \quad \text{and} \quad \frac{S_1}{S_2} = F \left(\frac{W_1}{W_2}\right)^2$$

where:

$(\frac{S_W}{S_O})$ = ratio of take-off distance in headwind to distance in a calm (Figure 13)

F = weight correction factor (Figure 14).

Because of the lack of good theoretical lift, drag, and thrust data a comparison of the two methods presented for estimating the take-off ground distance and the time to lift-off has been omitted.

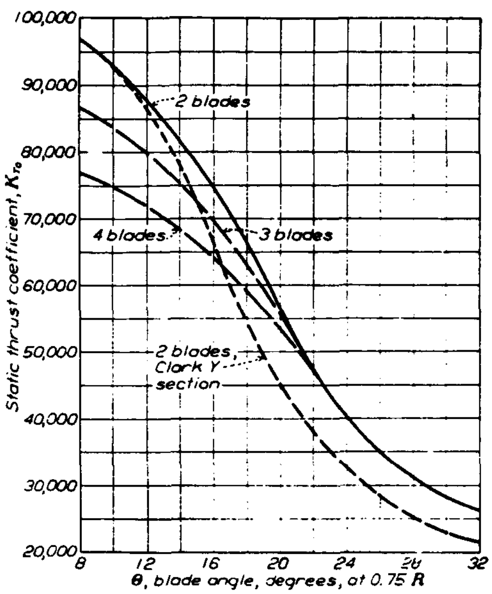


Figure 7. Static thrust coefficient versus θ.

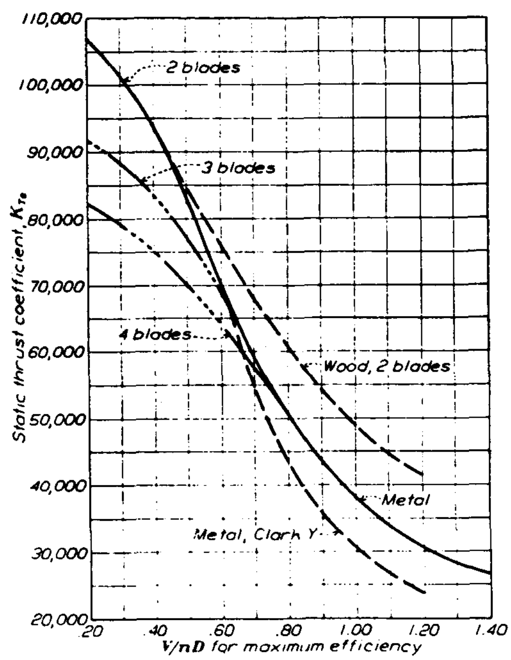


Figure 8. Static thrust coefficient versus V/nD.

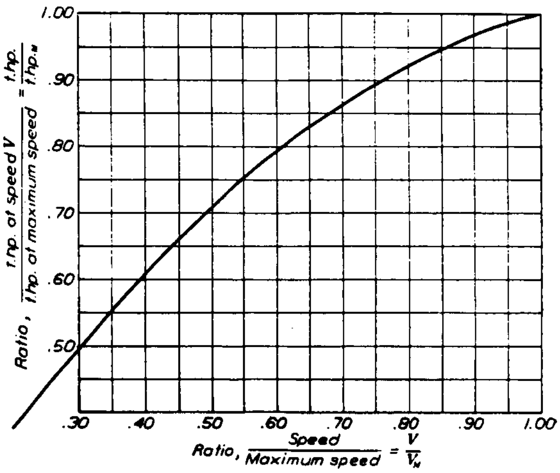


Figure 9. General full-throttle t.hp. curve.

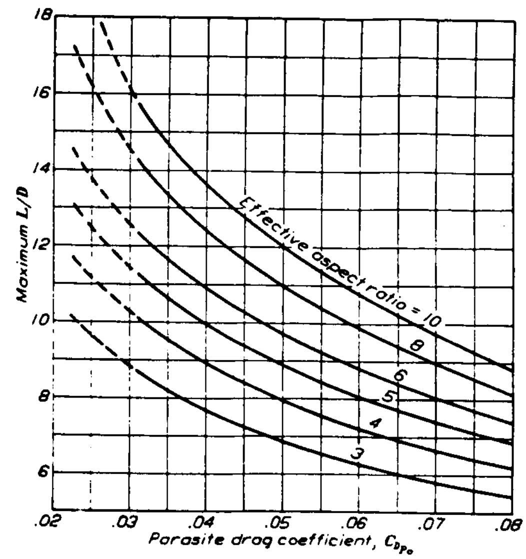


Figure 10. Maximum L/D versus parasite drag coefficient.

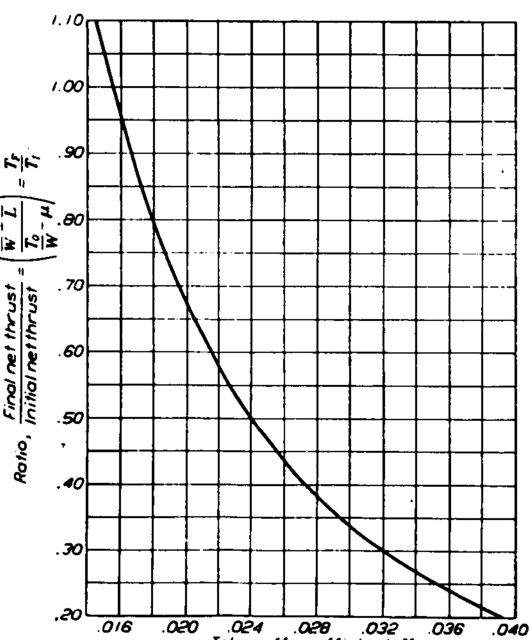


Figure 11. Take-off run in calm.

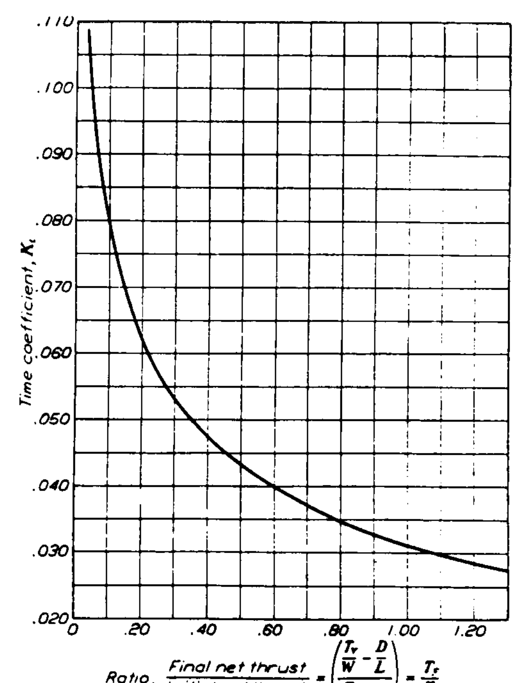


Figure 12. Coefficient for time to take-off.

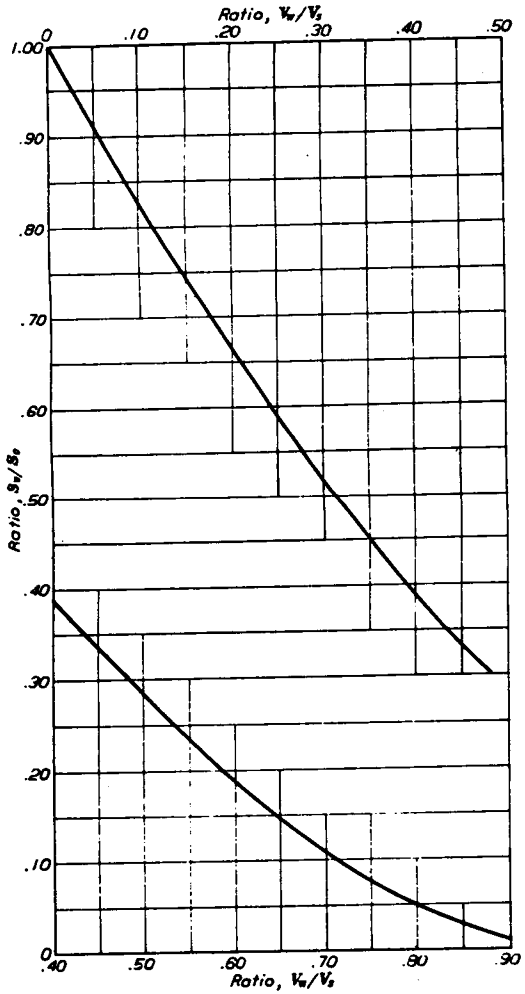


Figure 13. Effect of wind on take-off run

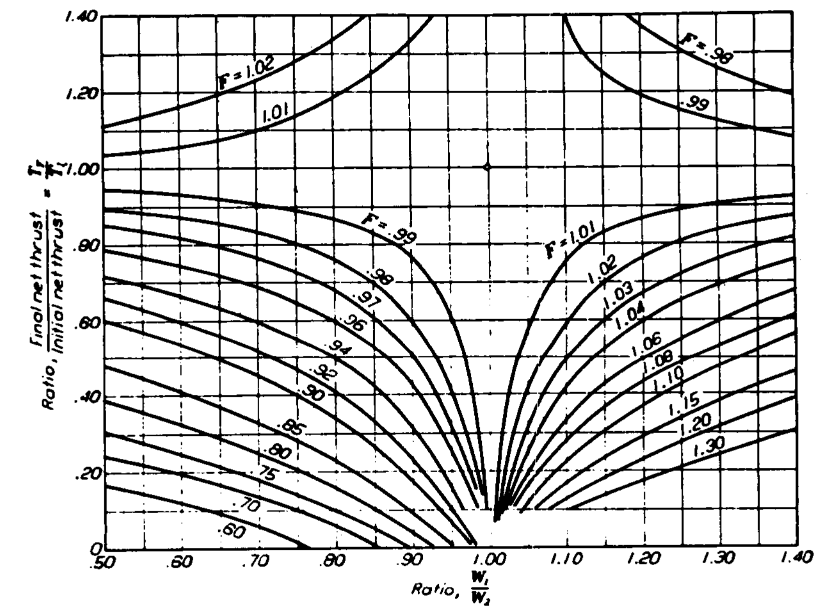


Figure 14. Effect of weight on take-off run.

Climb to 50 Feet

The approximate ground distance traveled in attaining an altitude of 50 feet after lift-off can be expressed by a relation similar to the one given for the ground run distance:

$$S = \int_{V_{LOF}}^{V_{50}} \frac{(V - V_w)}{\bar{a}}\, dt ,$$

or

$$S_{50} = \int_{V_{LOF}}^{V_{50}} \frac{W}{g} \frac{(V - V_w)dV}{[T - (C_D)\ \frac{1}{2}\ \rho_o \sigma V^2]} \tag{25}$$

where V_{50} is the velocity of the airplane at an altitude of 50 feet. The time required to attain an altitude of 50 feet can similarly be expressed as:

$$t_{50} = \int_{V_{LOF}}^{V_{50}} \frac{W}{g} \frac{dV}{[T - (C_D)\ \frac{1}{2}\ \rho_o \sigma V^2 S]} . \tag{26}$$

The total take-off distance traversed in going from a position of rest to an altitude of 50 feet is thus $S_G + S_{50}$, and the total take-off time is $t_G + t_{50}$. Equations (3), (4), (5), and (6) can be integrated numerically if the velocities, which are the limits of integration, are known; if C_D and C_L are considered independent of velocity, and if the functional form of T(V) is known. As an aid for estimating V_{LOF} and V_{50} it should be noted that Part 23 of the Federal Aviation Regulations (Ref. 30) requires that V_{50} be at least 1.3 times the zero thrust stall speed. One would also expect V_{LOF} to be approximately 1.1 times the stall speed.

Landing Approach

The landing approach can be divided into two parts--a steady-state glide path where the airplane is in the final landing configuration prior to touch-down and the flare.

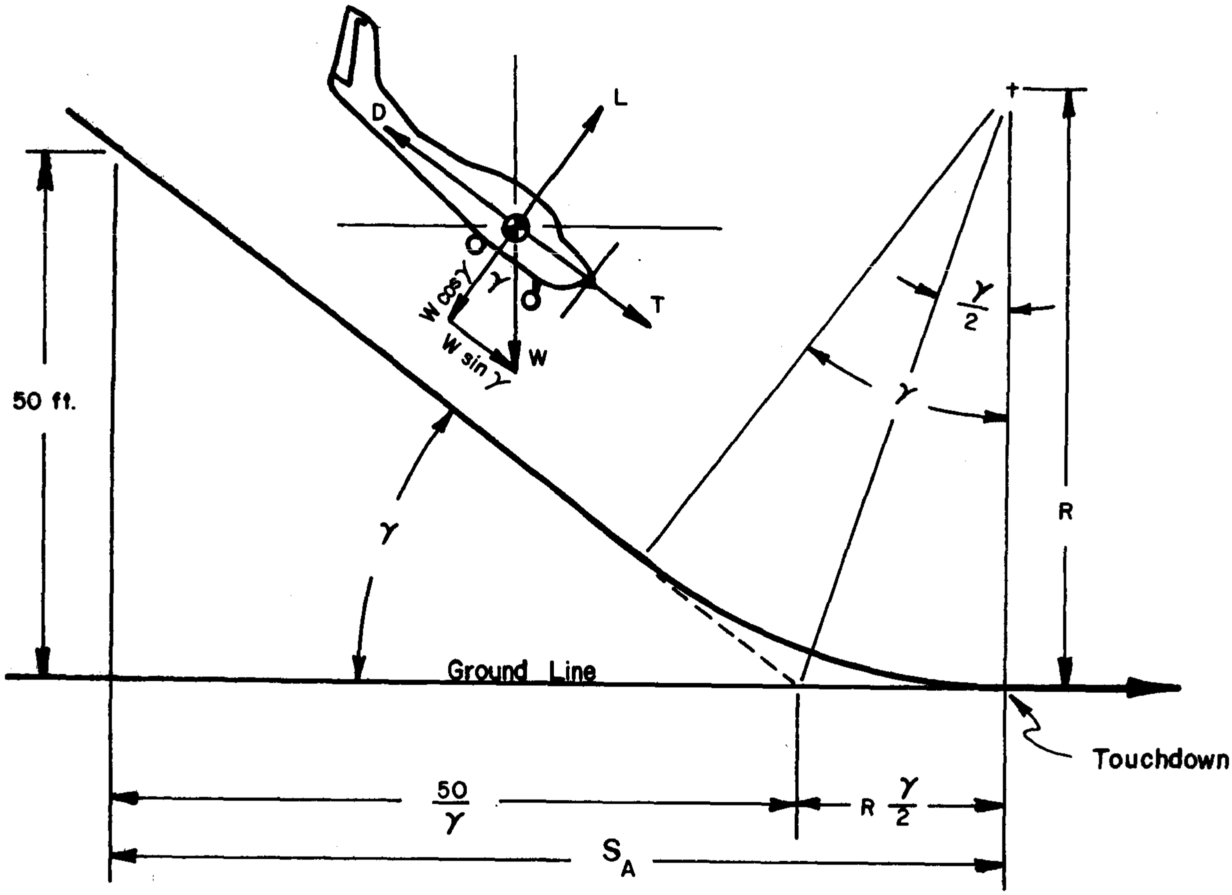


Figure 15. Typical pattern for landing approach, flare, and touchdown.

Assuming that the flare is a circular arc (Figure 15) and that γ is small, then S_A, the ground distance traversed in descending from an altitude of 50 feet to touchdown, can be expressed as:

$$S_A = \frac{50}{\gamma} + \frac{R\gamma}{2} \quad . \tag{27}$$

Since γ is small, V is assumed to be constant during the approach glide. An expression for γ and R must now be determined. With the weight approximately equal to the lift, γ can be written as

$$\gamma = \frac{C_D}{C_L} - \frac{T}{W} \quad . \tag{28}$$

The acceleration normal to the flight path needed to flare is attained by rotating the airplane to a higher C_L value, say C_L'. The lift force during the flare can be written

$$L' = C'_L \rho_o \frac{\sigma}{2} SV^2 \; .$$

Then the force normal to the flight path is

$$F_N = L' - W \cos \gamma = L' - W \; .$$

However, during the glide $L \simeq W$; thus $L'/W = C'_L/C_L$. The normal force can therefore be written as

$$F_N = W \frac{C'_L}{C_L} - W = W(n - 1) \tag{29}$$

where $n = C'_L/C_L$ = load factor (the maximum value of n which may be applied to C_L is dictated by stall or buffet limits).

The force normal to the flight path can also be expressed as

$$F_N = \frac{W}{g} a_N$$

where

$$a_N = \frac{V^2}{R} \; .$$

Equating the two expression for normal force yields the relation

$$W(n - 1) = \frac{W}{g} \frac{V^2}{R}$$

or

$$R = \frac{V^2}{g(n - 1)} \; . \tag{30}$$

But,

$$V^2 = \frac{L'}{S} \frac{2}{\rho_o \sigma C'_L} = \frac{nW}{S} \frac{2}{\rho_o \sigma C'_L} \; . \tag{31}$$

With equation (28), (30), and (31) and the fact that $C'_L = nC_L$, equation (27) can be expressed as

$$S_A = \frac{50}{\left(\frac{C_D}{C_L} - \frac{T}{W}\right)} + \frac{\frac{W}{S}\left(\frac{C_D}{C_L} - \frac{T}{W}\right)}{\rho_o \sigma g(n - 1) C_L} \; . \tag{32}$$

The ground distance for approach can thus be estimated if C_D, C_L, T, and the change in lift coefficient which occurs during the flare, n, are known.

Landing Ground Run

The landing ground run can be described as (1) a short ground run (approximately two seconds) immediately following touchdown while the airplane is being changed from landing configuration to braking configuration and (2) the remaining ground run which brings the airplane to a complete stop. The distance covered in landing transition is taken to be

$$S_{TRAN} = \frac{V_{TD} + V_B}{2} \Delta t_{TRAN} \tag{33}$$

where

V_{TD} = touchdown velocity*,

V_B = speed at full braking configuration,

Δt_{TRAN} = the transition time from touchdown to full braking (approximately 2 seconds).

The ground run braking distance is obtained in the same manner as the take-off ground run,

$$S_B = \int_{V_B}^{V_W} \frac{(V - V_W)}{\bar{a}} dV \tag{34}$$

where

V_B = initial braking speed,
V_W = headwind velocity,
$\bar{a}$ = deceleration rate.

The acceleration (deceleration if it is negative) term derived in the take-off section may be used here with an appropriate braking value of μ (a good approximate value of μ for aircraft tires on an asphalt runway is 0.3) Thus,

$$S_B = \int_{V_B}^{V_W} \frac{W(V - V_W)dV}{g[T - \mu_B W - (C_D - \mu_B C_L) \frac{\sigma}{2} V^2 \rho_O S]} \ . \tag{35}$$

In using the above equation the values of C_D and C_L must be those actually obtained in a landing run. For instance, if the flaps are retracted after touchdown then the approach C_L should not be used to calculate the braking distance. Similarly if an aircraft is equipped with reversible-pitch propellers, the thrust term should take this into account. If a reversible propeller is not used then T should represent the idle thrust.

If one desires to achieve the minimum braking distance then a large negative value of the acceleration term is desired. This is best obtained with a negative thrust (reversible propeller), a high drag coefficient, and a low value of C_L.

---

* It should be remembered that the touchdown velocity must be low enough that the lift will not be greater than the weight at the touchdown altitude.

PATH PERFORMANCE

DESCRIPTION OF THE PATH PERFORMANCE EQUATIONS AND THE INTEGRATION PROCEDURE EMPLOYED TO OBTAIN FLIGHT TIME HISTORIES

In recent years, development of the digital computer and improved numerical techniques have made possible the prediction of aircraft performance in a somewhat more sophisticated manner. Prior to this, the static analysis detailed in the preceding section was used exclusively in estimating aircraft performance. However, when more-refined analysis is desired the approach most often employed is integration of the vehicle equations of motion, a system of simultaneous, ordinary differential equations. This integration yields time histories of both vehicle and flight path parameters for various control inputs. By repeatedly solving these equations for diversified inputs, the practical optimum of some desired parameter (*i.e.*, range, rate of climb, . . .) may be approached.

The general equations governing aircraft performance as derived in Appendix B have been programmed for numerical solution. A discussion of the solution procedure and of some typical results for various input combinations constitutes the majority of the section. A detailed user guide, complete with an example case and a program listing, is presented in Appendix D.

The basic equations (B-19) are presented below for reference.

$$\dot{x} = V \cos \gamma \tag{36}$$

$$\dot{h} = V \sin \gamma \tag{37}$$

$$\dot{V} = \frac{gP}{WV} - \frac{g}{W}\frac{SV^2}{2} C_D(\alpha)\ \rho(h) - g \sin \gamma \tag{38}$$

$$\dot{\gamma} = \frac{g}{W}\frac{SV}{2} C_L(\alpha)\ \rho(h) - \frac{g}{V} \cos \gamma \tag{39}$$

$$\dot{W} = - cP \tag{40}$$

$$\rho(h) = \rho_o(1.0 - 6.86 \times 10^{-6}h)^{4.26} \tag{41}$$

$$C_L(\alpha) = C_{L_\alpha}\alpha + C_{L_{(\alpha=0)}} \tag{42}$$

$$C_D(\alpha) = C_{D_o} + k[C_L(\alpha)]^2 + k_1[C_L(\alpha)]^{k_2} \tag{43}$$

$$P \leq P_{max}(h,V) \tag{44}$$

The first equation, which relates the horizontal distance traveled to the velocity and flight path angle, is added to those of Appendix B to permit the direct calculation of range. Note that the fuel-flow rate is considered to be directly proportional to engine power rather than to thrust, since concern here is primarily for aircraft powered by piston engines. Also, the variation in thrust with angle of attack is restricted to small α, since the thrust vector is assumed to lie along the body axis.*

In programming the general equations, Equation (43) was simplified by setting k equal to zero, yielding the following functional form.

$$C_D(\alpha) = C_{D_O} + k_1[C_L(\alpha)]^{k_2} \tag{45}$$

When drag data are available, the three parameters in Equation (45) (C_{D_O}, k_1, k_2) may be found by the curve-fitting scheme presented in the previous section. A comparison of the accuracy obtained by fitting drag data with the three parameters of Equation (45) as opposed to the four parameters of Equation (43) indicates a negligible difference except for angles of attack approaching stall. Since most performance trajectories do not entail continued operation near the minimum speed, this simplification introduces no significant error. For calculations in which operation near the minimum velocity is of primary interest, the reader should investigate the procedure presented in the previous section which employs the four parameter fit to drag data.

If drag data are not accessible, the standard parabolic drag polar evolves from Equation (45) as follows:

$$C_D(\alpha) = C_{D_O} + \frac{C_L^2}{\pi e AR} \tag{46}$$

where

e = Oswald's efficiency factor
AR = wing aspect ratio
C_{D_O} = drag coefficient which is independent of angle of attack.

From a mathematical viewpoint, the performance problem is one of solving four nonlinear ordinary differential equations involving six dependent variables and one independent variable--time. Consequently, there are two degrees of freedom which imply an infinite number of trajectories for each set of initial conditions. To obtain a unique trajectory, time histories of six unknowns must be specified. Although there are fifteen possible combinations of these unknowns taken two at a time, the programmed solution procedure allows the user to specify only fourteen sets of inputs since weight and power can not be specified independently. The optimal desired

---

* These restrictions are not fundamental to the technique. More general expressions, if available, may easily be substituted without significant increase in computing time.

(range, rate of climb, etc.) strongly influences the selection of variables to be specified. The variation of any chosen variable and its derivatives is assumed known throughout the entire trajectory. Initial conditions for each of the differentiated variables must be provided for use in starting the integration procedure.

The integration technique, essentially a Runge-Kutta Adams-Bashforth predictor-corrector procedure, was altered on the basis of a paper by Charles Treanor (Ref. 31). Since it is self-starting, the modified Runge-Kutta was used to calculate the first four points. Then the modified predictor-corrector performed the remaining integration until the step size was either increased or decreased due to some error criterion.

The following is a brief synopsis of the evolution of the integration procedure from the classical Runge-Kutta into its final form. The modification proposed by Treanor is applicable to any differential equation in which the derivative of the dependent variable has a strong dependence on the difference between the value of the variable and that of some slowly changing function. This type behavior is notably evident in calculating the derivative of flight path angle as can be seen by grouping the right hand side of Equation (39) as follows:

$$\dot{\gamma} = \frac{g}{V} \left(\frac{\rho(h)V^2S}{2} C_L\right) \frac{1}{W} - \frac{g}{V} \cos \gamma \; . \tag{47}$$

Substituting for the lift and rearranging yields:

$$\dot{\gamma} = - \frac{g}{V} \left[\cos \gamma - \frac{L}{W}\right] \; . \tag{48}$$

Obviously, the derivative is strongly dependent upon γ and the slowly changing lift to weight ratio. When solving a system of equations using standard Runge-Kutta Adams-Bashforth technique, the above equation induces enormous oscillations which severely restrict the size of the integration increment. The Treanor modification to the classical Runge-Kutta method removes these oscillations in any integration interval where this strong dependence is detected. For regions not exhibiting this particular behavior, the modified method reverts to classical fourth order Runge-Kutta.

However, even with the enlarged integration step size permitted by the modified Runge-Kutta, solution of the system of equations proved to be very costly (in computer time) when integrating trajectories of long duration (*i.e.*, eight hours or more). This is because Runge-Kutta procedures essentially obtain the solution twice for each integration interval. In a particular interval the results are first computed at $t_2 = t_1 + \Delta t$ and then recomputed using steps of $t/2$. A comparison between the two results indicates the accuracy obtained and determines whether the integration step size should be increased or decreased. Thus the equations are actually solved twice, once in normal step sizes and again in half steps with a comparison following the larger step. This computational complexity suggests the use of a fourth order predictor-corrector such as that of Adams-Bashforth (Ref. 32). A modification of this predictor-corrector, which can accommodate derivatives strongly related to the value of the dependent variable and which provides the compatibility conditions necessary for use with the modified Runge-Kutta procedure, is derived in Appendix G. Use of this combined integration procedure permitted the step size to increase by one to three orders of magnitude depending upon the type and nature of the specified variables. Computational times were so drastically reduced that integration of eight to ten hour flight trajectories were relatively inexpensive. Execution times for an eight hour trajectory may vary from about nine seconds, for a solution having altitude and angle of attack specified as constant, to slightly more than two minutes for a trajectory with a superimposed oscillatory mode which results from specification of angle of attack as a constant and power as the maximum available.

The constraint on the general performance problem, Equation (44), demands that the power for any flight maneuver always be less than or equal to the maximum power available at a particular altitude and velocity. To fulfill this requirement, the maximum power available as a function of altitude and velocity must be determined for the aircraft under investigation. The calculation of $P_{max}(h,V)$ for use in this general integration procedure is detailed in the previous section. Briefly, the technique utilizes a spline fit of several power available versus velocity points at some reference altitude to predict the maximum power available at any altitude and velocity. The program compares the actual power calculated with $P_{max}(h,V)$ at each integration step to insure that the resulting trajectory satisfies the maximum power available constraint.

TYPICAL RESULTS FROM INTEGRATION OF THE PATH PERFORMANCE EQUATIONS

The performance of a vehicle in actual flight may exhibit many features of interest in addition to the equilibrium maximums predicted by static calculations. Some aircraft experience accelerations too severe for reliable static approximation. For example, in a zoom maneuver or a rapid climb, the airframe dynamics are obviously nonlinear and their description demands a more sophisticated treatment. Large weight changes associated with high power loadings also emphasize the need for this type analysis. The following examples serve to illustrate this treatment and will, perhaps, encourage the reader's use of the more realistic path performance approach possibly employing the accompanying computer program (Appendix D).

The following examples all employ the classical parabolic drag polar. However, computation with the three parameter polar of Equation (45) proceeds with equal ease. A value for C_{D_O} was taken from Reference 1 and c, the specific fuel comsumption, was given an average value of 0.6 lbs/hp-hr.

First, the determination of aircraft range for several different sets of specified parameters was investigated. When considering the prediction of maximum range for a given amount of fuel, the investigator who draws upon his previous experience immediately considers angle of attack as one of the specified variables, since for quasi-steady conditions flying at $\alpha = \alpha_{(L/D)_{max}}$ produces the greatest range per pound of fuel. Calculations carried out using the path performance equations (and discussed below) prove the validity of this result for light aircraft. The second specified variable could be any of several whose effect on the resulting trajectory might yield an increased range. To determine $\alpha_{(L/D)_{max}}$ for the three parameter drag polar of Equation (45), first form the ratio (C_L/C_D) and differentiate with respect to C_L, set the derivative equal to zero, then solve for $C_{L_{(L/D)_{max}}}$. This process yields

$$C_{L_{(L/D)_{max}}} = \left[\frac{C_{D_o}}{k_1(k_2 - 1)}\right]^{1/k_2} . \qquad (49)$$

For the parabolic drag polar, $k_2 = 2$, and

$$C_{L_{(L/D)_{max}}} = \left(\frac{C_{D_o}}{k_1}\right)^{1/2} \qquad (50)$$

or

$$C_{D_i} = C_{D_o} \qquad (\text{at } \alpha = \alpha_{(L/D)_{max}}) . \qquad (51)$$

Inserting (50) into Equation (42) yields α for maximum lift to drag ratio. Each trajectory began with similar initial conditions and was terminated upon consumption of 222 pounds of fuel. Initial and final values for several pairs of specified variables are given in Table 5.

	Case 1		Case 2		Case 3		Case 4		Case 5	
Parameter	Initial Value	Final Value	Initial Value	Final Value	Initial Value	Final Value	Initial Value	Final Value	Initial Value	Final Value
t, minutes	0.0	480.3	0.0	380.3	0.0	460.0	0.0	472.0	0.0	384.0
h, feet	10,000.	10,000.	10,000.	10,000.	10,000.	12,780.	10,000.	10,000.	10,000.	10,000.
V, feet/second	149.0	142.6	177.9	170.2	151.0	151.0	149.0	142.8	149.0	142.7
γ, degrees	0.0	0.0	0.0	0.0	0.0	.03834	0.0	- 3.88	0.0	- 3.9
α, degrees	5.8778	5.8778	3.0	3.0	5.8778	5.8778	5.8778	5.8778	5.8778	5.8778
C_L	.7817	.7817	.5503	.5503	.7817	.7817	.7817	.7817	.7817	.7817
C_D	.0538	.0538	.04023	.04023	.0538	.0538	.0538	.0538	.0538	.0538
W, pounds	2650	2428	2650	2428	2650	2428	2650	2428	2650	2428
P, horsepower	49.4	43.3	62.6	54.9	50.07	48.9	49.4	0.0	111.3	0.0
x, miles	0.0	795.5	0.0	752.0	0.0	788.6	0.0	794.5	0.0	795.0
Description of Specified Variables:	$\alpha = \alpha_{(L/D)_{max}}$ $= .10258$ rad $= 5.8778°$		$\alpha =$ constant $= .05236$ rad $= 3.0°$		$\alpha = \alpha_{(L/D)_{max}}$ $= .10258$ rad $= 5.8778°$		$\alpha = \alpha_{(L/D)_{max}}$ $= .10258$ rad $= 5.8778°$		$\alpha = \alpha_{(L/D)_{max}}$ $= .10258$ rad $= 5.8778°$	
	h = constant = 10,000 ft		h = constant = 10,000 ft		V = constant = 151 ft/sec		$P=P_{req}\left(\frac{\sigma-.165}{\sigma_{ref}-.165}\right)$ $=(27173)\left(\frac{\sigma-.165}{.5734}\right)$		$P = P_{max}(V,h)$	

Table 5. Results of several test cases designed to obtain maximum range.

The specified variables for Case (1) are constant angle of attack and constant altitude. During the eight-hour trajectory, the weight decreased by 222 pounds with an associated velocity drop of 6.4 feet per second and a corresponding power reduction of about 6 hp. In Case (2) a smaller angle of attack (α = 3 degrees) was specified and the other parameters remained as in Case (1). After expending 222 pounds of fuel, the range was 43 miles less than for Case (1). This illustrates the advantage of flying with angle of attack for maximum lift to drag ratio. In Case (3), angle of attack and a slightly larger velocity than that resulting from previous trajectories were specified. This attempt to further range through a slight increase in speed proved futile, since the vehicle climbed nearly 2,000 feet and then began to glide after using the allotted fuel until at the same final altitude it obtained an equivalent range.

In Case (4) the power was specified as the minimum required for level flight--at 10,000 feet with $\alpha = \alpha_{(L/D)_{max}}$--multiplied by a factor to allow for density variation as the vehicle climbs due to fuel usage. Once the weight reached 2,428 pounds (222 pounds of fuel burned), the power was set equal zero and the vehicle permitted to glide until it descended to 10,000 feet altitude. A similar power schedule was followed in Case (5) with the exception that power was assigned to be the maximum available as a function of local altitude and velocity. As before, when fuel consumption reached 222 pounds, the power was set equal zero with the effect being a glide descent to 10,000 feet. Figure 16 presents a comparison of power schedules with the resulting altitude and velocity variations. The vehicle of Case (4) glided the last six miles; whereas, the Case (5) craft finished by gliding for nearly 45 miles. However, as Table 5 indicates, both trajectories produced the same range, but Case (5) required nearly 1.5 hours less flight time.

Motivation produced by the performance optimization study of Reference 33 led to the investigation of aircraft range using a combination of three power settings. The trajectory was divided into three sections--ascent, cruise, and descent--with a different power setting corresponding to each regime. First, the power was designated as the maximum available for the ascent stage; then at some predetermined altitude the power was reset so that during cruise the thrust and drag were equal; finally, when the assigned portion of fuel (common to each trajectory) was consumed, the power was shut off and the vehicle executed a glide descent back to its initial altitude. Since range was of primary concern, the second variable was specified as $\alpha = \alpha_{(L/D)_{max}}$. A family of trajectories was integrated with the altitude for switching the power from maximum to that for thrust equal drag being varied from 14,000 feet to 22,000 feet in increments of 2000 feet. Figure 17 presents altitude and power variations corresponding to the upper and lower switching altitudes. After descent to 10,000 feet, the range for every trajectory was equivalent. Since these flights were only five hours in length, the final range was compared with Case (1) of Table 5 at the five hour mark and they were also equivalent. The major conclusion from the above investigation of aircraft range is that most trajectories will produce ranges of equal magnitude whenever one of the specified variables

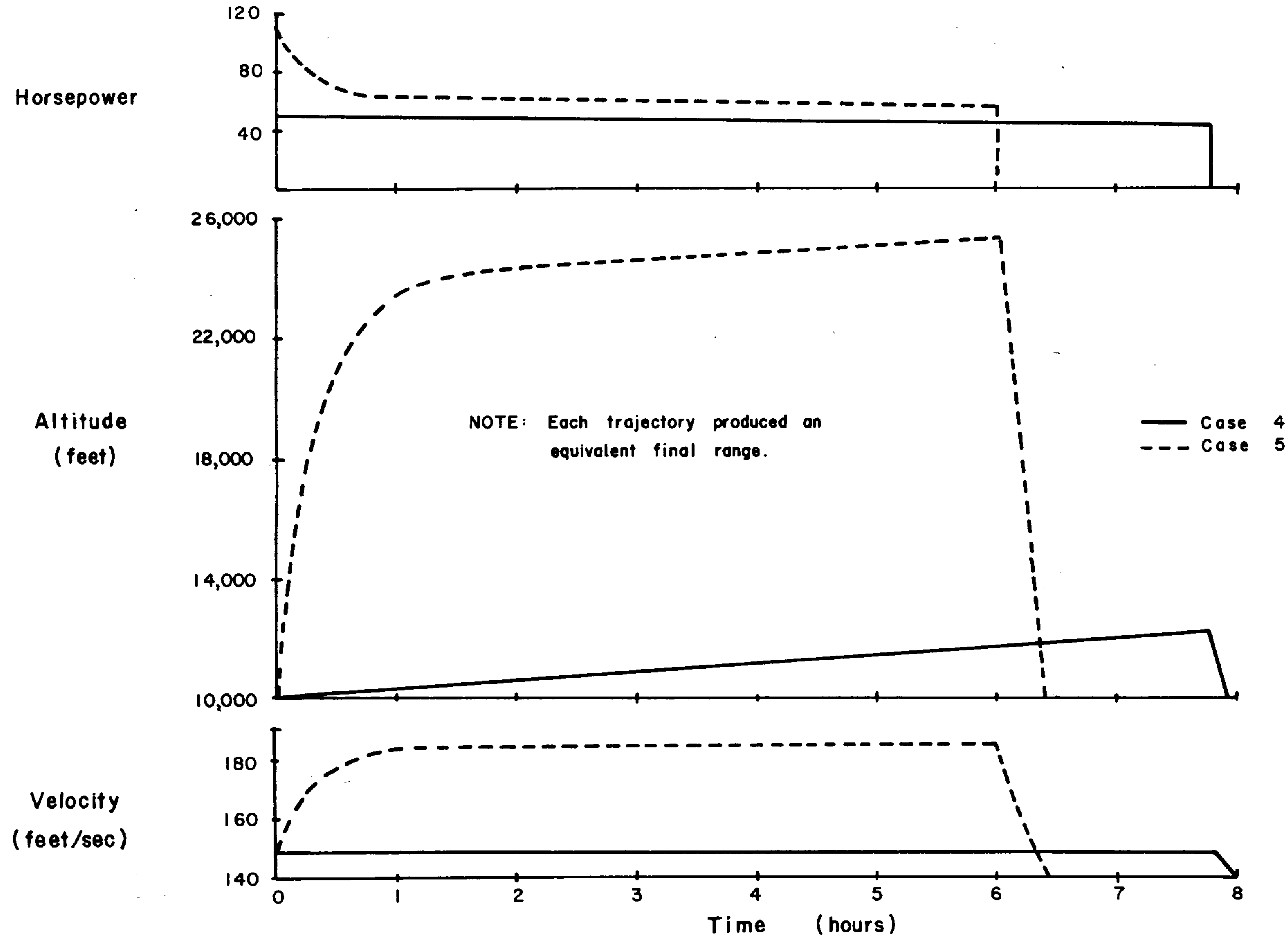


Figure 16. Time histories of several parameters resulting from analysis of maximum range.

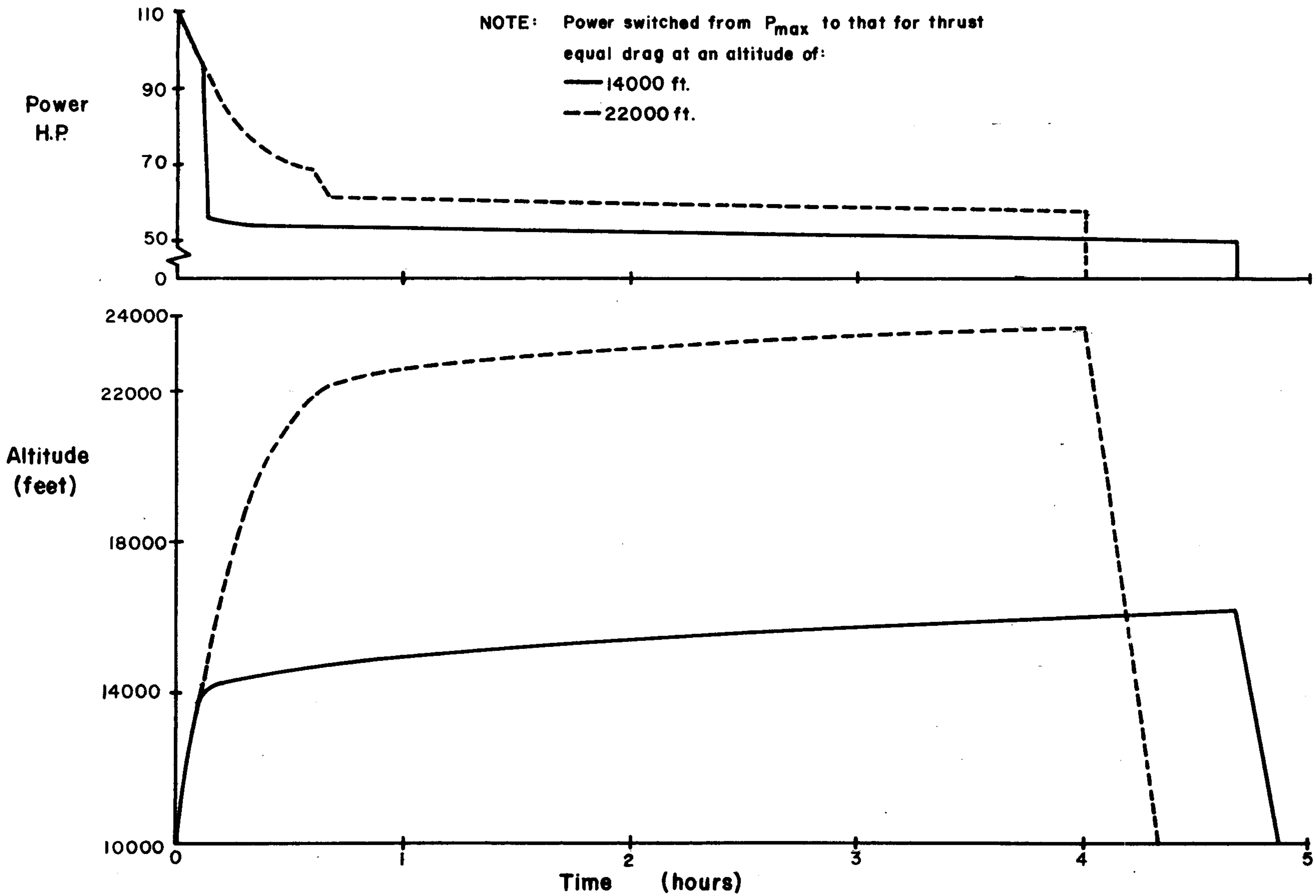


Figure 17. Time histories of power and altitude for trajectories using three different power settings.

is set as $\alpha = \alpha_{(L/D)_{max}}$.* Thus the major objective for most pilots desiring maximum range should be flight at approximately the angle of attack for maximum lift to drag ratio. To obtain more realistic results than those in Table 5 (*i.e.* 795 miles on 222 pounds of fuel is quite an exaggeration) and to accurately approximate $\alpha_{(L/D)_{max}}$, improvement in the prediction of three-dimensional lift and drag characteristics is mandatory. More precise knowledge of the variation of fuel flow rate with power required, aircraft speed, and altitude, and engine speed and throttle setting is also necessary. Acceptable trends and desired objectives may be formulated using the classical lift and drag relations; however, realistic values from performance calculations will result only when high quality aerodynamic and engine test data or sophisticated prediction techniques are utilized to estimate the body forces and the fuel flow rate.

Since the objective of the preceding analysis was determination of parameter behavior throughout the flight, large time scales of several hours or more were employed. These large time divisions, however, often mask some interesting behavior which occurs simultaneously but can be observed only on a small time scale. An inset, describing the first two minutes of flight path angle variation with time for the trajectory of Case (5), is presented in Figure 18.

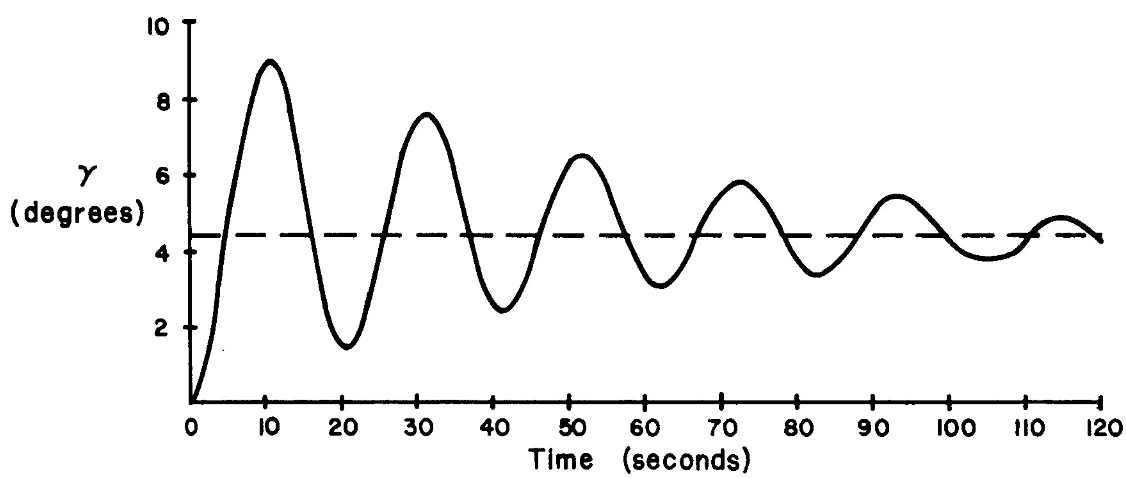


Figure 18. Sample phugoid trajectory.

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* This result should not be too surprising. Consider for the moment the situation which would exist if the drag were zero. Then the energy required to traverse any trajectory between point A and point B would be the same, namely zero. Because of the existence of drag, energy can be dissipated in two ways: that required to overcome friction in steady flight and that associated with the increasing disorder (entropy) of accelerated motion. Now, if the drag is relatively small to begin with and the accelerations are also modest, then it is reasonable to expect that a great many different flight trajectories will have very similar energy requirements, that is, they will require the same amount of fuel plus or minus a pound or two. One would expect very different results for trajectories involving near sonic speeds with significant accelerations.

Since this behavior normally results whenever angle of attack (*i.e.* lift coefficient) is held constant, the oscillation observed in Figure 18 is commonly known as a phugoid trajectory. The period of oscillation, about 21 seconds, is typical of the phugoid motion for light aircraft. The oscillations are damped as the velocity increases and the vehicle climbs, due to the decrease in amount of excess power available. Observation of this trajectory on a larger time scale, reveals only an average γ which decreases from about 4.4 degrees to approximately zero late in the flight. Recovery of the aforementioned phugoid motions from the total trajectory symbolizes the very general nature of this solution and emphasizes the lack of most restrictive assumptions.

The application of a general solution technique to landing flight appears very informative. For analysis, the landing in subdivided into two portions, the approach and the flare, with velocity and flight path angle specified in each interval. A basic geometrical description of the glide path and flare is presented in Figure 19.

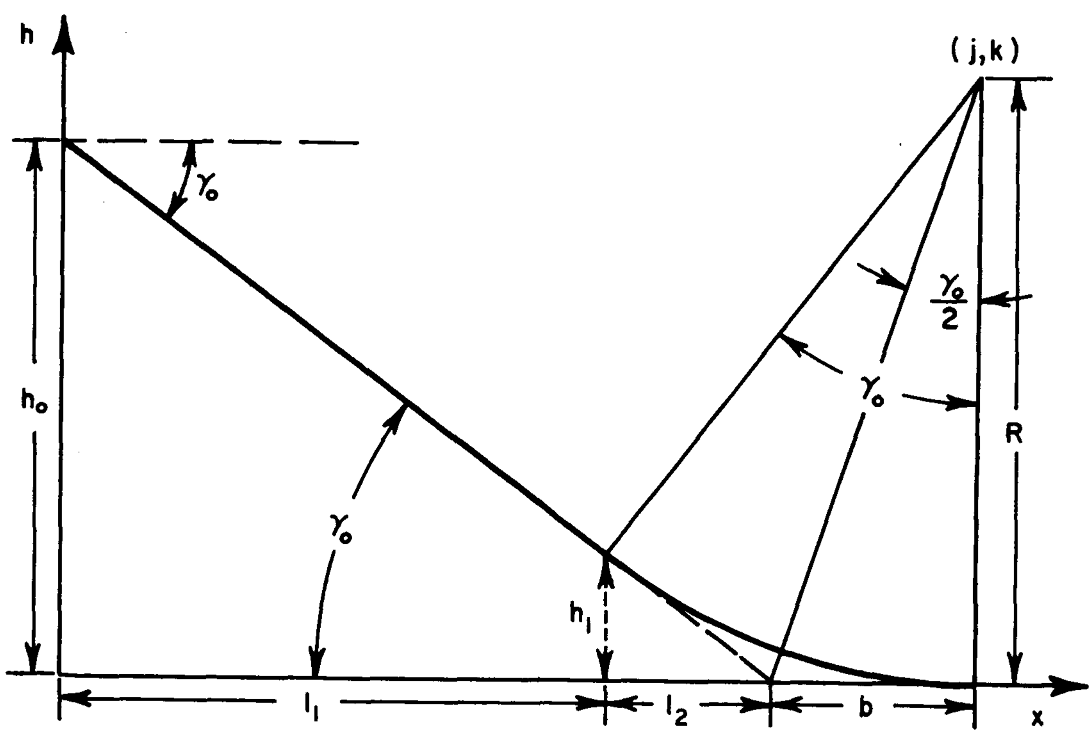

Figure 19. Geometrical description of landing analysis.

The overall concept is first to descend at constant velocity and flight path angle until reaching an altitude of h_1 and then execute a flare with velocity decreasing and γ becoming zero at touchdown. For illustration, the flight path angle is specified so as to produce a circular arc flare and velocity is linearly decreased from the approach speed to some touchdown velocity. Specification of γ in the flare region necessitates finding the equation of a circle of radius R with center at (j, k) passing through points (ℓ_1, h_1) and $(\ell_1 + \ell_2 + b, 0)$. This equation is of the form:

$$(x - j)^2 + (h - k)^2 = R^2 \quad . \tag{52}$$

Since

$$\tan \gamma = \frac{dh}{dx} = - \frac{(x - j)}{(h - k)} \tag{53}$$

flight path angle for the flare regime is given by:

$$\gamma = - \tan^{-1} \left(\frac{x - j}{h - k}\right) \quad . \tag{54}$$

For calculation of j and k, values of h_o (altitude at beginning of approach), γ_o (constant glide path angle during approach), and either h_1 (altitude of flare initiation) or b (distance from intersection of extended approach path with ground to the point of touchdown) must be given. Using these three parameters, values for j and k may be determined from the following formulae:

$$h_1 = b \sin \gamma_o \tag{55}$$

$$R = b/\tan(\gamma_o/2) \tag{56}$$

$$\ell_1 = (h_o - h_1)/\tan \gamma_o \tag{57}$$

$$k = R \tag{58}$$

$$j = \ell_1 + \sqrt{h_1 (2R - h_1)} \quad . \tag{59}$$

Thus velocity and flight path angle may be specified as follows:

$$\gamma = \begin{cases} - \gamma_o & h_1 < h \leq h_o \\ - \tan^{-1} \left(\frac{x - j}{h - k}\right) & 0 \leq h \leq h_1 \end{cases} \tag{60}$$

$$V = \begin{cases} V_o & h_1 < h \leq h_o \\ (V_o - V_T)\left(\frac{h}{h_1}\right) + V_T & 0 \leq h \leq h_1 \end{cases} \tag{61}$$

where

$$V_o = \text{approach velocity}$$
$$V_T = \text{touchdown velocity.}$$

Also,

$$\dot{\gamma} = \begin{cases} 0.0 & h_1 < h \le h_o \\ \dfrac{(h - k)}{(h - k)^2 + (x - j)^2} \left[\dfrac{(x - j)}{(h - k)} \dot{h} - \dot{x}\right] & 0 \le h \le h_1 \end{cases} \quad (62)$$

$$\dot{V} = \begin{cases} 0.0 & h_1 < h \le h_o \\ \left(\dfrac{V_o - V_T}{h_1}\right) \dot{h} & 0 \le h \le h_1 \end{cases} \quad (63)$$

Consider the following example for which:

$$\begin{aligned} h_o &= 1500 \text{ feet} \\ \gamma_o &= 2.5 \text{ degrees} = 0.04363 \text{ radians} \\ b &= 400 \text{ feet} \\ V_o &= 140 \text{ feet/sec} \\ V_T &= 90 \text{ feet/sec} \quad . \end{aligned} \quad (64)$$

Substituting these in Equations (55) - (59) yields:

$$\begin{aligned} h_1 &= 17.45 \text{ feet} \\ \ell_1 &= 33{,}958.57 \text{ feet} \\ k &= 18{,}333.1 \text{ feet} \\ j &= 34{,}758.19 \text{ feet} \quad . \end{aligned} \quad (65)$$

Evaluating Equation (62) indicates $\dot{\gamma}$ is small enough to be assumed zero everywhere. The resulting time histories for this example are presented in Figure 20. At touchdown the rate of descent is less than 0.2 feet/second and the flight path angle is approximately zero. For a more complete landing analysis, the investigator might consider the effect of various approach and touchdown velocities; the effect of several other glide descent

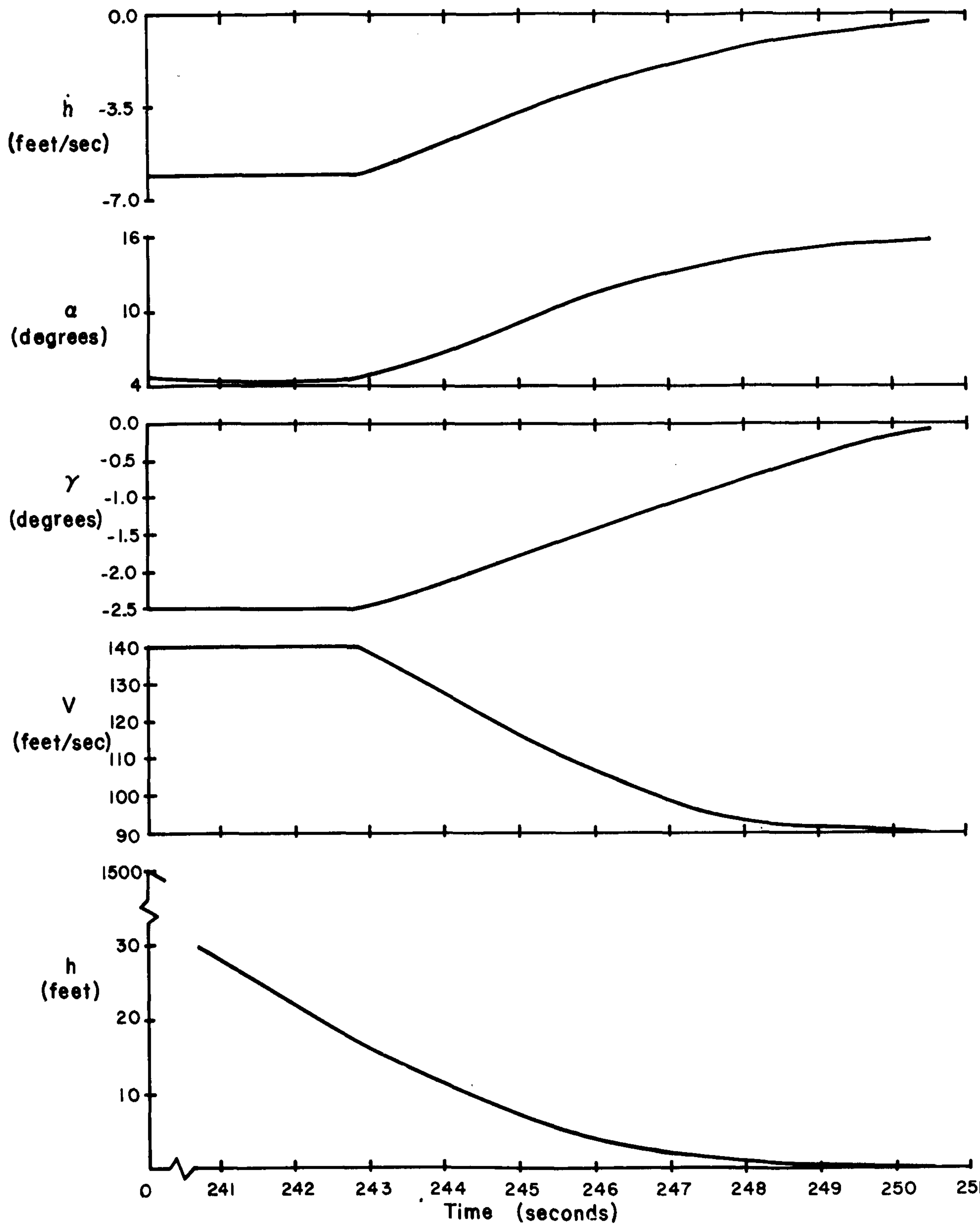

Figure 20. Time histories of several parameters governing an example landing.

angles and possibly, how the vehicle responds for flare at different altitudes. These possibilities indicate the flexibility with which this solution technique may be employed to conduct a landing analysis or a study of other performance criteria involving maneuvers.

Since static predictions are only special cases of the general integrated solutions, it is interesting to compare the results obtained by the two methods. By specifying variables in a manner similar to the assumptions made for derivation of the static equations, the computational accuracy and the effects of weight changes can be assessed.

First, the maximum vehicle speed at any altitude is calculated by simply specifying the altitude as a constant and power as the maximum available. The integration is terminated once the velocity has settled to a near constant value. Results are presented in Figure 21 for various altitudes.

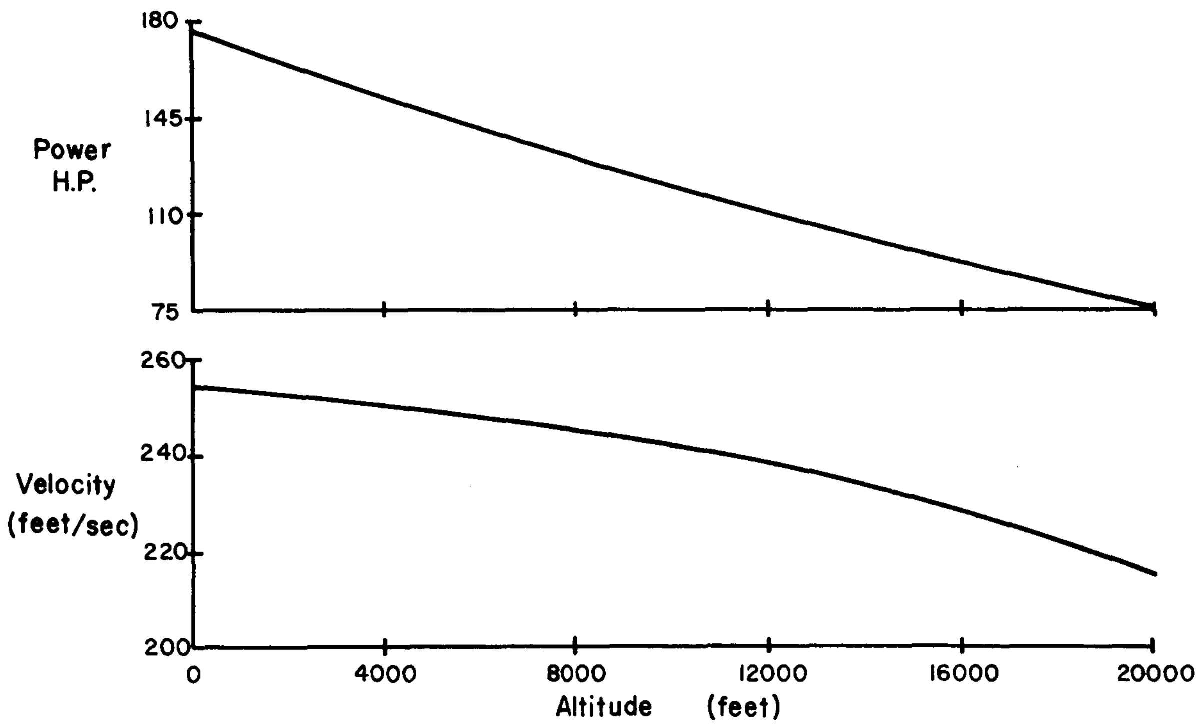

Figure 21. Determination of maximum velocity as a function of altitude.

At sea level, V_{max} was found to be 254 feet per second which agrees well with the static approximations in Table 1. This same result could have also been obtained by specifying power as P_{max} and using a family of curves for different constant velocities. The maximum velocity would be the entry for which the altitude remained constant ($\dot{h} = 0$). For a velocity greater that V_{max}

(level flight) the vehicle would dive and a velocity less than V_{max} would induce a climb.

Next, minimum power and the velocity for P_{min} were determined at a particular altitude. The specified variables were: (1) h equal a constant and (2) V as a constant incremented throughout a plausible range. Each constant velocity solution produces only one value for P_{min} in Figure 22.

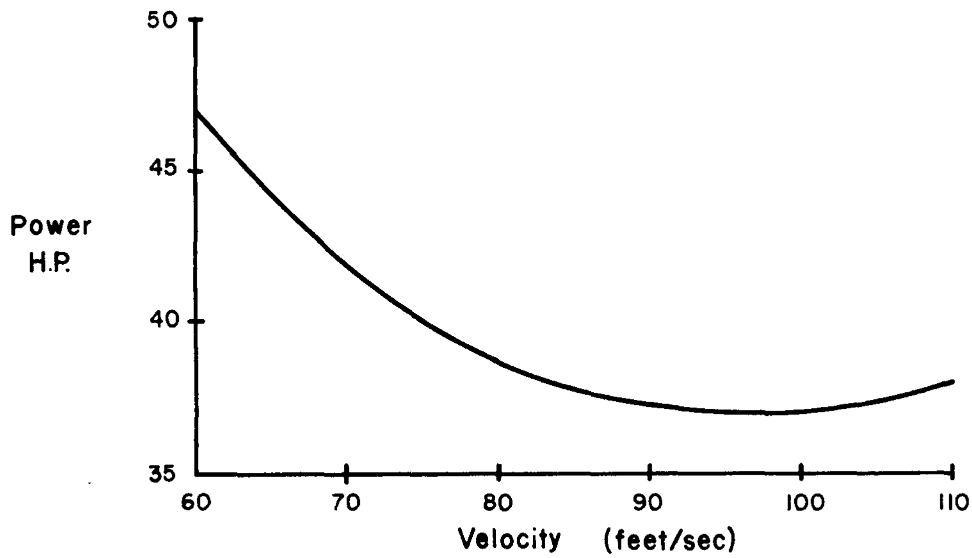

Figure 22. Determination of P_{min} and speed for P_{min}.

The results are for sea level altitude, but the process is easily repeated at other altitudes. As Figure 22 indicates, the minimum power was about 37.0 hp (20,350 ft-lbs/sec) with a corresponding velocity of 97.0 feet per second. These results compare favorably with those of Table 1.

The maximum climb angle (γ_{max}) and the velocity for γ_{max} were also calculated. Power was specified as the maximum available and velocity was made constant for each of several trajectories. The maximum climb angle corresponding to a particular flight path and its specified velocity is then plotted on Figure 23. The overall γ_{max} is then selected to be the largest of those in Figure 23. Its associated velocity is then termed as the velocity for maximum climb angle. Even though the maximum flight path angle (13.15 degrees) occurs at 82 feet per second, any velocity from 65 to 110 feet per second will produce a γ_{max} within one degree of the absolute maximum. Thus, the maximum climb angle is somewhat insensitive to velocity changes over a rather wide speed range. Even though, this is considered to be the maximum climb angle, larger flight path angles may be obtained for short periods of time from an oscillatory climb similar to the phugoid motion described by Figure 18. However, an average over these oscillations

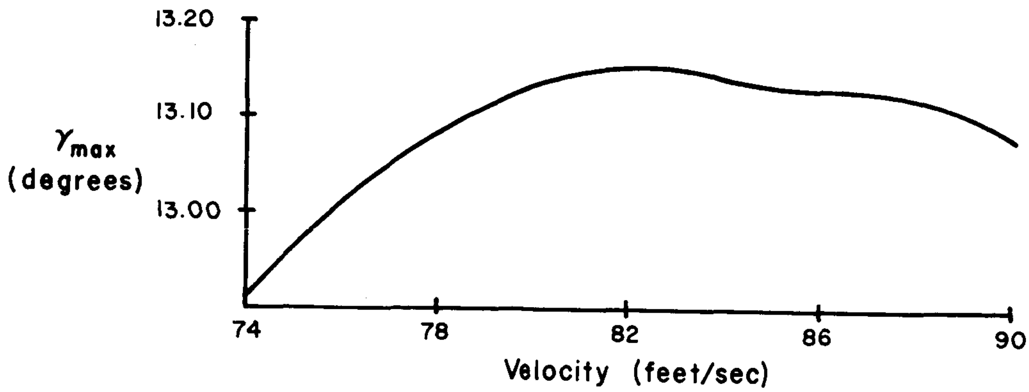

Figure 23. Determination of γ_{max} and speed for γ_{max}.

always yields, at most, a mean climb angle with magnitude equal to the γ_{max} from Figure 23. Point performance estimates are similar as indicated in Table 1.

To determine the minimum velocity at an altitude, power was specified as the maximum available and velocity was again set equal a different constant for each of several paths. The initial altitude is designated as the height of interest and the equations are integrated for several minutes to determine whether the vehicle ascends ($\dot{h}$ positive) or descends ($\dot{h}$ negative). The velocity for zero rate of climb is then the minimum velocity for the altitude of interest. Figure 24 presents calculation of V_{min} at sea level. This value of V_{min} agrees well with that of Table 1 obtained from point performance using a parabolic drag polar. Table 2 illustrates a more realistic minimum speed corresponding to an improved drag polar.

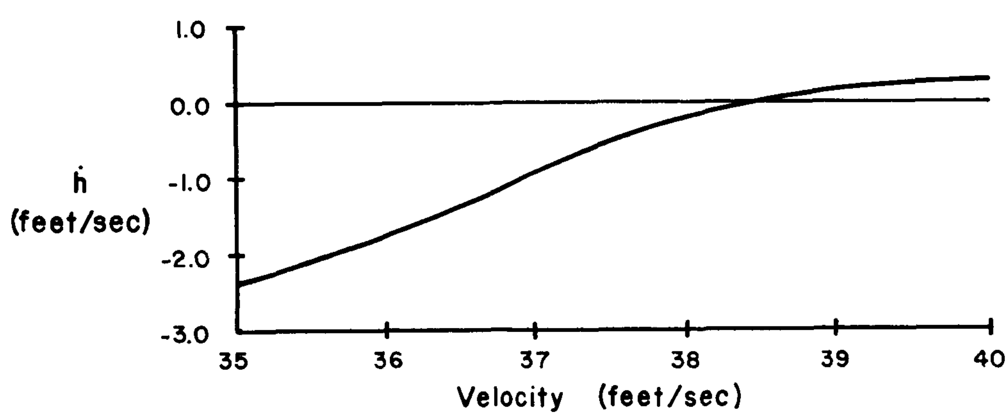

Figure 24. Calculation of minimum velocity for level flight.

Finally, calculations were made of absolute and service ceilings. Power was specified as the maximum available. A family of 90 minute trajectories was calculated with each entry corresponding to a different specified constant angle of attack. This is not meant to imply that the primary objective was to obtain absolute and service ceilings as a function of angle of attack. Instead, α was merely a convenient parameter for creating a family of curves from which to choose the maximums. Two altitude points from each trajectory were recorded in Figure 25. The first was the altitude at which $\dot{h}$ = 1.67 ft/sec and the second was the final height after integrating over 90 minutes of real time. The peak in the lower curve of Figure 25 is termed the service ceiling and the maximum of the upper curve is called the absolute ceiling. Since in actuality the vehicle will continue to climb slowly as fuel is consumed, absolute and service ceiling are associated with a particular weight. The trajectories represented by Figure 25 used approximately 42 pounds of fuel in the 90 minute trajectory. The increase of 100 to 150 feet in these values of absolute and service ceiling as compared to those of Table 1 is a consequence of the weight reduction. The velocities corresponding to the peak points of Figure 25 are 153.6 ft/sec for the service ceiling and 158.0 ft/sec at absolute ceiling. These were likewise increased due to the weight reduction.

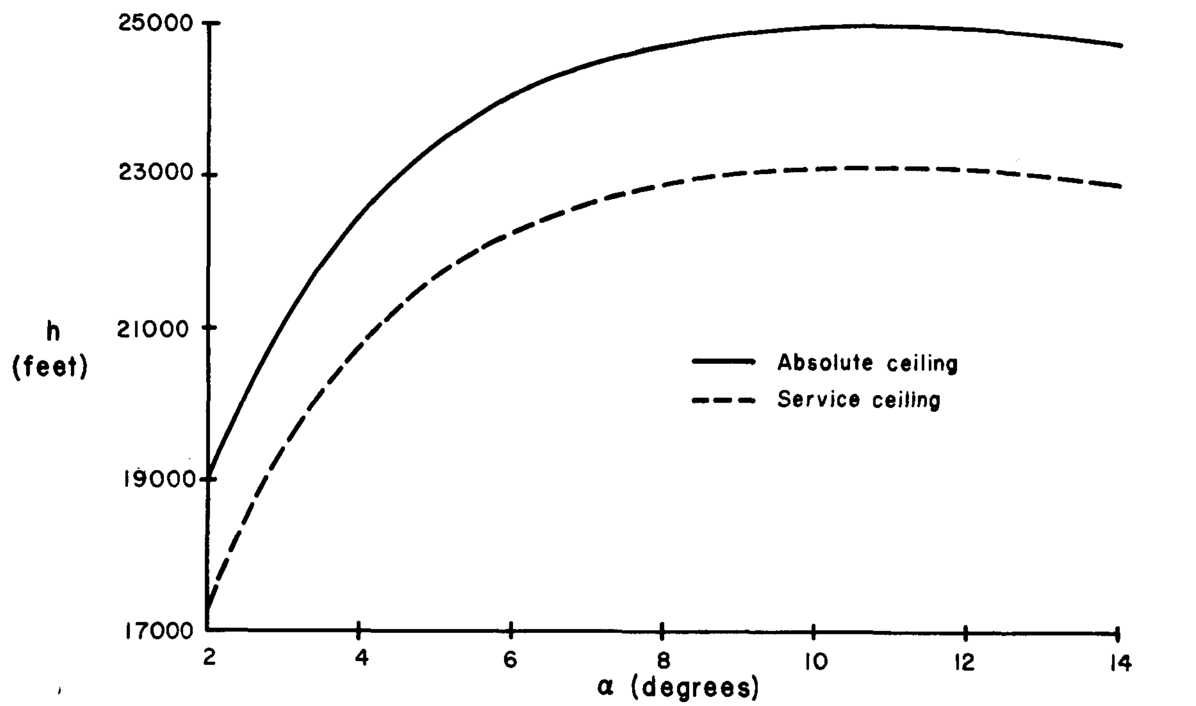

Figure 25. Determination of absolute and service ceilings.

The preceding discussion of calculation procedures for recovery of several static performance parameters indicates that velocity for γ_{max} is not critical and discrepancies in other parameters are related to weight variations.

The foregoing examples have demonstrated that if desired the path performance method can be used to recover the usual static performance parameters. In addition, this method can be used to determine the performance of the aircraft during accelerating flight or in response to a complex schedule of pilot control inputs.

In passing, some comments may be made regarding range performance at "practical speeds", *i.e.*, those obtained at high percents of rated horsepower. Table 5 shows that when $\alpha < \alpha_{L/D_{max}}$ the range is reduced. In the example given $\alpha = 3°$ rather than $5.85°$, α for $(L/D)_{max}$. In this case, the range is about 5% less. This calculation assumes that the specific fuel consumption is the same at all power settings and engine speeds. As noted in Appendix H the specific fuel consumption may be as much as 1.4 times as high at rated power as at the power required for maximum range. Thus, not only does the aircraft operate more inefficiently but so does the engine. By dropping back to about 75% power the engine can be made to operate very near maximum efficiency so that under these conditions one must contend only with aerodynamic inefficiencies. A calculation not discussed above was performed for $P = P_{max}$, $h = 10,000$ ft. at the same specific fuel consumption as used in the construction of Table 5. The range was only 63% of that achieved by flying at $\alpha = \alpha_{L/D_{max}}$ so that at 75% power one would expect at least a 25% decrease in maximum range. At maximum power one could reasonably expect a 55% decrease in maximum range.

CONCLUDING REMARKS

The estimation of many light aircraft performance characteristics has, as a result of the computer programs provided herewith, been reduced to a quick, inexpensive procedure. The programs require that the user supply estimates or experimental measurements of the lift and drag variations with angle of attack and the variation of power into the airstream as a function of speed and altitude. The drag data need not be parabolic. Any accurate set of experimental points can be fit precisely by the program and utilized in the estimation. Similarly, the program can also fit and utilize arbitrary power data. As presently written the program assumes linear variations of C_L with angle of attack and fuel flow with power, but these restrictions are not inherent in the method. A knowledgeable user can modify the programs without undue difficulty so that they will accept any type of variation he might wish to employ. The authors would be pleased to provide suggestions on how this might be accomplished to interested users.

The path performance programs provided with this work offer the user an opportunity to study many facets of the flight of light aircraft in addition to the equilibrium performance parameters. While the programs do not provide a rigorous method of finding the optimum path performance, they are sufficiently inexpensive to use that they can usually be employed by one with some understanding of the physical situation to arrive at paths indistinguishable from the optimal through a trial and error process. It is felt that substantial flight time could be saved in evaluating new designs by using analytical solutions to indicate the paths of greatest interest.

Ultimately, one would like to be able to specify just the vehicle geometry and its powerplant and have a program which will take this information and compute the vehicle's performance. By repeating the process with some discretion it will be possible to determine a configuration for optimum performance subject to the usual constraints of costs, fabrication techniques, passenger size, etc. Development of the necessary methods and programs for carrying out this procedure is planned for the near future. The programs presented in Reference 2 can then be used to compute the stability and riding qualities of the optimum performance configuration. If necessary, the configuration can be altered and the process repeated until the desired characteristics are achieved.

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APPENDICES

APPENDIX A - Nomenclature

AR	aspect ratio
a, $\bar{a}$	acceleration
BHP	engine brake horsepower
C_{1_j}, C_{2_j}, C_{3_j}, C_{4_j}	coefficients for Spline curve-fit procedure
C_D	drag coefficient
C_{D_i}	induced drag coefficient
C_{D_o}	zero-lift drag coefficient
C_L	lift coefficient
C_L'	lift coefficient after landing flare
$C_{L(L/D)_{max}}$	lift coefficient for maximum lift to drag ratio
$C_{L(\alpha=0)}$	lift coefficient for zero angle of attack
c	specific fuel consumption
D	drag or propeller diameter
e	Oswald's efficiency factor
F	force or take-off weight correction factor
g	acceleration due to gravity (32.2 ft/sec^2)
h	altitude
$\dot{h}$	aircraft rate of climb
K_t	take-off time coefficient
K_s	take-off ground run coefficient
K_{T_o}	static thrust coefficient
k_1, k_2, k_3, k_4	coefficients of a general drag polar
L	lift
L'	lift after the landing flare

M	mass
N	load factor (C_L'/C_L) or engine speed in revolutions per minute
n	engine speed in revolutions per second or load factor (C_L'/C_L)
P	power actually put into the airstream, sometimes called thrust horsepower, *i.e.*, thrust × velocity
R	radius of circular arc traversed in descending from an altitude of 50 feet to touchdown
RPM	engine speed in revolutions per minute
S	wing area or distance
T	thrust or minimum time to climb
t	time
thp_m	maximum thrust horsepower
V	velocity
W	weight
x	horizontal distance traveled
α	angle of attack
$\alpha_{(L/D)_{max}}$	angle of attack for maximum lift to drag ratio
β	sideslip angle
Δt	increment in time
γ	flight path angle in the vertical-horizontal plane (see Figure B-1)
η	propeller efficiency
μ	coefficient of friction
π	3.14159
ρ	density of air
ρ_o	density of air at sea level
σ	ρ/ρ_o, ratio of density of air at an altitude to the density at sea level

ϕ	flight path angle in the vertical-co-normal plane (see Figure B-1)
ψ	flight path angle in the horizontal-co-normal plane (see Figure B-1)

Subscripts:

A	approach
B	braking
G	ground run
LOF	lift-off
max	maximum
min	minimum
N	normal to flight path
s	take-off
TD	touchdown
w	wind
zL	zero lift
50	an altitude of 50 feet
o	take-off

A dot over a quantity denotes the time derivative of that quantity.

APPENDIX B - Derivation of General Performance Equations

Those undertaking the estimation of aircraft performance characteristics for the first time will usually find it instructive to review the theoretical basis of the methodology. If one considers that the word performance connotes such things as "how fast will it go?", "how long will it take to get to 10,000 feet", etc., he realizes that he is asking questions about the motion of the aircraft under certain constraints. The classical means of describing the motion of a rigid body in space is through solutions of the equations of motion, mathematical statements of Newton's Second Law of Motion. It is not surprising that the same equations are also used to study the stability of the motions, the problem usually termed the dynamics of the airplane. The study of aircraft performance, then, can be considered as one view of the general problem of aircraft motions while the study of stability and control simply views the same problem in another light. The difference in the two views, as will become clear from the development below, is primarily one of time scale. Stability and control analysis is concerned with transient disturbances from an equilibrium motion. The disturbances of interest usually have periods of less than 30 seconds. Performance analysis on the other hand, is concerned primarily with quasi-equilibrium flight. Further, the period of interest generally exceeds 30 seconds. Thus, the following development proceeds in roughly the same fashion as the development of the stability equations. The points of departure and difference are noted.

To begin, make three assumptions:

Assumption I. The earth and its atmosphere are flat and non-rotating. There is no motion of the atmosphere with respect to the earth. Accelerations measured with respect to a Cartesian coordinate system fixed in the earth's atmosphere are therefore true accelerations in inertial space.

Assumption II. The motion of interest is that of the aircraft's center of gravity only. Other motions, for example rotation about the center of gravity, are of interest only insofar as they affect the translational motion of the center of gravity.

Assumption III. While the mass of the aircraft does not remain constant with time the change in momentum associated with the fuel mass ejected with the engine exhaust may be neglected in computing the motion of the aircraft.

As a consequence of these assumptions one may describe the aircraft as a point mass and its motion by three equations:

$$\Sigma F_x = Ma_x$$

$$\Sigma F_y = Ma_y \qquad \text{(B-1)}$$

$$\Sigma F_z = Ma_z \quad .$$

This is in contrast to stability analysis where the motions of the aircraft about its center of gravity are of great interest and require the inclusion of the body's three rotational degrees of freedom. On the other hand, the time frame treated by stability analysis is sufficiently short that the mass can be considered to be constant.

We take as our coordinate system an x-axis parallel with the earth and pointed in the direction of motion at the beginning of the time of interest; a y-axis pointed to the right of the direction of motion; and a z-axis pointed straight down. In contrast with stability analysis where the axis system rotates with the airframe this axis system, once chosen, remains fixed with time. In this system the aircraft velocity has three components:

$$\dot{x} = V \cos \gamma \cos \psi$$

$$\dot{y} = V \sin \psi \cos \gamma \qquad \text{(B-2)}$$

$$\dot{h} = - V \sin \gamma \quad .$$

γ and ψ are flight path angles as defined in Figure B-1.

Substition of Equations (B-2) into (B-1) yields

$$\Sigma F_x = M (\dot{V} \cos \gamma \cos \psi - V\dot{\gamma} \sin \gamma \cos \psi - V\dot{\psi} \cos \gamma \sin \psi)$$

$$\Sigma F_y = M (\dot{V} \sin \psi \cos \gamma + V\dot{\psi} \cos \gamma \cos \psi - V\dot{\gamma} \sin \gamma \sin \psi) \qquad \text{(B-3)}$$

$$\Sigma F_z = M (-\dot{V} \sin \gamma - V\dot{\gamma} \cos \gamma) \; .$$

Assumption IV. The forces acting on the aircraft are lift, drag, thrust, and weight. The lift acts normal to the flight path, the thrust* and drag are

---

* Assuming the thrust to act along the flight path is equivalent to assuming that the angle of attack is always small. It may therefore seem strange that the analysis employs expressions which provide accurate representations of the lift and drag to large angles of attack. This practice is justified because (1) light aircraft generally have insufficient power to operate continuously at very high angles of attack, (2) light aircraft are usually powered by piston engines and unshrouded propellers which makes it difficult to assert that the thrust always acts parallel to the wing chord, particularly at large angles of attack and (3) it is a simple matter to substitute expressions such as $T' \cos \alpha$ for T and $L' + T' \sin \alpha$ for L in the equations without materially affecting the method of solution or the computation time. The effect of power at angle of attack is treated in considerable detail in section 5.0 of Reference 13.

parallel to the flight path, and the weight acts along the positive z-axis. There is never a side force. This implies that the aircraft is symmetrical and is never yawed with respect to the flight path (β is always zero).

With this assumption, the left hand side of Equations (B-3) can be written

$$
\begin{aligned}
\Sigma F_x &= (T - D) \cos\gamma \cos\psi - L (\sin\gamma \cos\phi \cos\psi + \sin\psi \sin\phi) \\
\Sigma F_y &= (T - D) \sin\psi \cos\gamma - L (\sin\psi \sin\gamma \cos\phi - \sin\phi \cos\psi) \\
\Sigma F_z &= (T - D) \sin\gamma - L \cos\phi \cos\gamma + \frac{W}{g} \ .
\end{aligned}
\tag{B-4}
$$

ϕ is a flight path angle also defined in Figure B-1.

Combining (B-3) and (B-4), one obtains

$$
\begin{aligned}
&(T - D) \cos\gamma \cos\psi - L (\sin\gamma \cos\phi \cos\psi + \sin\psi \sin\phi) = \\
&\qquad M (\dot{V} \cos\gamma \cos\psi - V\dot{\gamma} \sin\gamma \cos\psi - V\dot{\psi} \cos\gamma \sin\psi) \\
&(T - D) \sin\psi \cos\gamma - L (\sin\psi \sin\gamma \cos\phi - \sin\phi \cos\psi) = \\
&\qquad M (\dot{V} \sin\psi \cos\gamma + V\dot{\psi} \cos\gamma \cos\psi - V\dot{\gamma} \sin\gamma \sin\psi) \\
&- (T - D) \sin\gamma - L \cos\phi \cos\gamma + W = \\
&\qquad M (-\dot{V} \sin\gamma - V\dot{\gamma} \cos\gamma) \ .
\end{aligned}
\tag{B-5}
$$

These equations provide the most general description of the motion of an aircraft acted upon by L, D, T, and W. Although the aircraft is thought of as a point mass, the magnitudes of the lift and drag forces are considered to depend upon α, the inclination of the aircraft to the flight path.

Assumption V. Motions of interest lie entirely within the x-z plane. ϕ and ψ are therefore zero.

This of course ignores possible interest in turning flight; however, as a consequence of Assumption V, Equations (B-5) simplify easily to

$$
\begin{aligned}
(T - D) \cos\gamma - L \sin\gamma &= M (\dot{V} \cos\gamma - V\dot{\gamma} \sin\gamma) \\
- (T - D) \sin\gamma - L \cos\gamma + W &= \frac{W}{g} (-\dot{V} \sin\gamma - V\dot{\gamma} \cos\gamma) \ .
\end{aligned}
\tag{B-6}
$$

Multiplying the first equation of (B-6) by cos γ and the second by -sin γ gives

$$
\begin{aligned}
&(T - D)\cos^2\gamma - L\sin\gamma\cos\gamma = \frac{W}{g}(\dot{V}\cos^2\gamma - V\dot{\gamma}\sin\gamma\cos\gamma) \\
&(T - D)\sin^2\gamma + L\sin\gamma\cos\gamma - W\sin\gamma = \frac{W}{g}(\dot{V}\sin^2\gamma + V\dot{\gamma}\cos\gamma\sin\gamma)\ .
\end{aligned}
\tag{B-7}
$$

Adding the two equations one obtains

$$
(T - D) - W\sin\gamma = \frac{W}{g}\dot{V}\quad . \tag{B-8}
$$

Multiplying the first equation of (B-6) by sin γ and the second by cos γ yields

$$
\begin{aligned}
&(T - D)\cos\gamma\sin\gamma - L\sin^2\gamma = \frac{W}{g}(\dot{V}\cos\gamma\sin\gamma - V\dot{\gamma}\sin^2\gamma) \\
&-(T - D)\sin\gamma\cos\gamma - L\cos^2\gamma + W\cos\gamma = \frac{W}{g}(-\dot{V}\sin\gamma\cos\gamma - V\dot{\gamma}\cos^2\gamma)
\end{aligned}
\tag{B-9}
$$

Adding these two equations, one obtains

$$
\frac{W}{g}V\dot{\gamma} = L - W\cos\gamma. \tag{B-10}
$$

Equation (B-8) and (B-10) are just simpler equivalents of (B-6). By writing the lift and drag forces in the equations in terms of the usual aerodynamic quantities and the thrust in terms of the power which is more appropriate for piston engine aircraft, (B-8) and (B-10) become finally

$$
\dot{V} = \frac{gP}{WV} - \frac{g}{W}C_D(\alpha)\cdot\rho(h)\cdot\frac{SV^2}{2} - g\sin\gamma \tag{B-11}
$$

$$
\dot{\gamma} = \frac{g}{W}C_L(\alpha)\cdot\rho(h)\cdot\frac{SV}{2} - \frac{g}{V}\cos\gamma\ . \tag{B-12}
$$

The reader will now observe that we have chosen to describe quasi-steady aircraft motions by two first-order, non-linear, ordinary differential equations. The dependent variables in these equations are P, V, γ, W, α, and h, while the independent variable is time. Thus to obtain a determinant system, one must supply four additional, independent relationships involving the dependent variables. One such relationship follows immediately from a definition of the rate of climb:

$$
\dot{h} = V\sin\gamma\quad . \tag{B-13}
$$

Assumption VI. The fuel flow rate is directly proportional to the power developed by the engine. While not strictly true, most engines have a region around cruise power where the specific fuel consumption is nearly constant. The propeller efficiency under these conditions is also nearly constant.

As a result of this assumption one may write a second auxillary relationship:

$$\dot{W} = - cP \quad . \tag{B-14}$$

If necessary, c can be generalized. For turbojet aircraft the fuel flow is approximately proportional to thrust output.

Unfortunately, no other general relationships among the dependent variables are known. It is therefore necessary to specify *a priori* time histories of two of the dependent variables in order to obtain unique solutions to the system of equations. This situation is familiar to pilots who realize that the aircraft's trajectory is dependent upon the variation of elevator position (speed) and power setting with time. The pilot also recognizes that changing the weight and operating altitude also changes the power setting and elevator angle one must employ to fly a given path or, conversely, for a given power setting and elevator angle, it changes the path one flies. With six variables there are 15 different combinations one may use to provide the two additional constraints needed. In view of Equation (B-14), however, power and weight cannot be specified independently. This leaves a total of 14.

Equations (B-11) and (B-12) contain in addition three implicit functions which must be provided if numerical values are to be obtained. The function $\rho(h)$ is of course the variation of density with geometric altitude. This is taken as

$$\rho(h) = \rho_o(1.0 - 6.86 \cdot 10^{-6}h)^{4.26}. \tag{B-15}$$

The functions $C_L(\alpha)$ and $C_D(\alpha)$ depend for their values upon the aircraft whose trajectories are desired. Although a third order or higher polynomial would be required to represent $C_L(\alpha)$ for all α from $- C_{L_{max}}$ to $+ C_{L_{max}}$, it is usually adequate to employ

$$C_L(\alpha) = C_{L_\alpha}(\alpha - \alpha_{zL}) \tag{B-16}$$

for all speeds above 1.2 $V_{STALL_{FLAPS\ UP}}$. Since the system of equations must be solved by a forward integration technique in any event, it does not add significantly to the overall computational complexity to choose a higher order function to represent $C_L(\alpha)$ if this should appear desirable. Similar comments can be made with respect to $C_D(\alpha)$. It is usually represented by

$$C_D(\alpha) = C_{D_o} + k\,[C_L(\alpha)]^2 \tag{B-17}$$

although more elaborate descriptions may be used. The addition of a term $k_1[C_L(\alpha)]^{k_2}$ will permit one to represent the drag coefficient at high angles of attack approaching stall quite accurately.

One final practical constraint must also be observed: the power required during any portion of the flight cannot exceed the maximum power available for that speed and altitude. Thus if one of the specified variables is not power one must always test the computed power required to insure that

$$P \le P_{max}\ (h,V)\ . \qquad \text{(B-18)}$$

Equations (B-11) through (B-18), collected below for convenience,

$$\begin{aligned}
\dot{V} &= \frac{gP}{WV} - \frac{g}{W}\frac{SV^2}{2} C_D(\alpha) \cdot \rho(h) - g \sin \gamma \\
\dot{\gamma} &= \frac{g}{W} C_L(\alpha) \cdot \rho(h) \cdot \frac{SV}{2} - \frac{g}{V} \cos \gamma \\
\dot{h} &= V \sin \gamma \\
\dot{W} &= - cP \\
\rho(h) &= \rho_o\ (1.0 - 6.86 \cdot 10^{-6} h)^{4.26} \\
C_L(\alpha) &= C_{L_\alpha}(\alpha) + C_{L_{\alpha=0}} \\
C_D(\alpha) &= C_{D_o} + k\,[C_L(\alpha)] + k_1[C_L(\alpha)]^{k_2} \\
P &\le P_{max}\ (h,V)
\end{aligned} \qquad \text{(B-19)}$$

plus the initial conditions of all six dependent variables provide a complete description of the motion of the aircraft c.g. in response to a set of forcing functions. The forcing functions will take the form of time histories of any two dependent variables, for example V and h or P and α. These may be chosen arbitrarily, but if one wishes to obtain solutions which represent the optimum performance it is necessary that he optimize the form of the two variables most appropriate to the parameter being determined. Unfortunately, it is not possible to prove that the two time histories chosen do in fact optimize the particular performance parameter. Only by comparing the solutions of (B-19) obtained by applying various physically meaningful and realizable time histories of the two most appropriate variables can a practical optimum be demonstrated. This solution procedure is discussed in detail in the path performance section.

It may be remarked in passing that while the procedure outlined above for determining how to fly the aircraft to obtain the best possible value of each performance parameter of interest is the most rigorous known, it is practical only with the use of a large, high-speed digital computer. It is for this reason that the so-called static or point performance parameters came into general use in the years before the availability of electronic computers. These procedures, developed from Equations (B-19) assuming that the dependent variables <u>do not change with time</u>, are discussed in detail in the static performance section. Note, however, that whereas the first two equations of (B-19) are differential equations, the static performance equations are all algebraic and therefore much easier to solve. Even so, evaluation of some of the static performance parameters involves solution of a quartic equation or even simultaneous solution of a pair of quartic equations. Traditionally, such problems have been solved graphically. In the present work the problems are solved numerically through the use of digital computer programs.

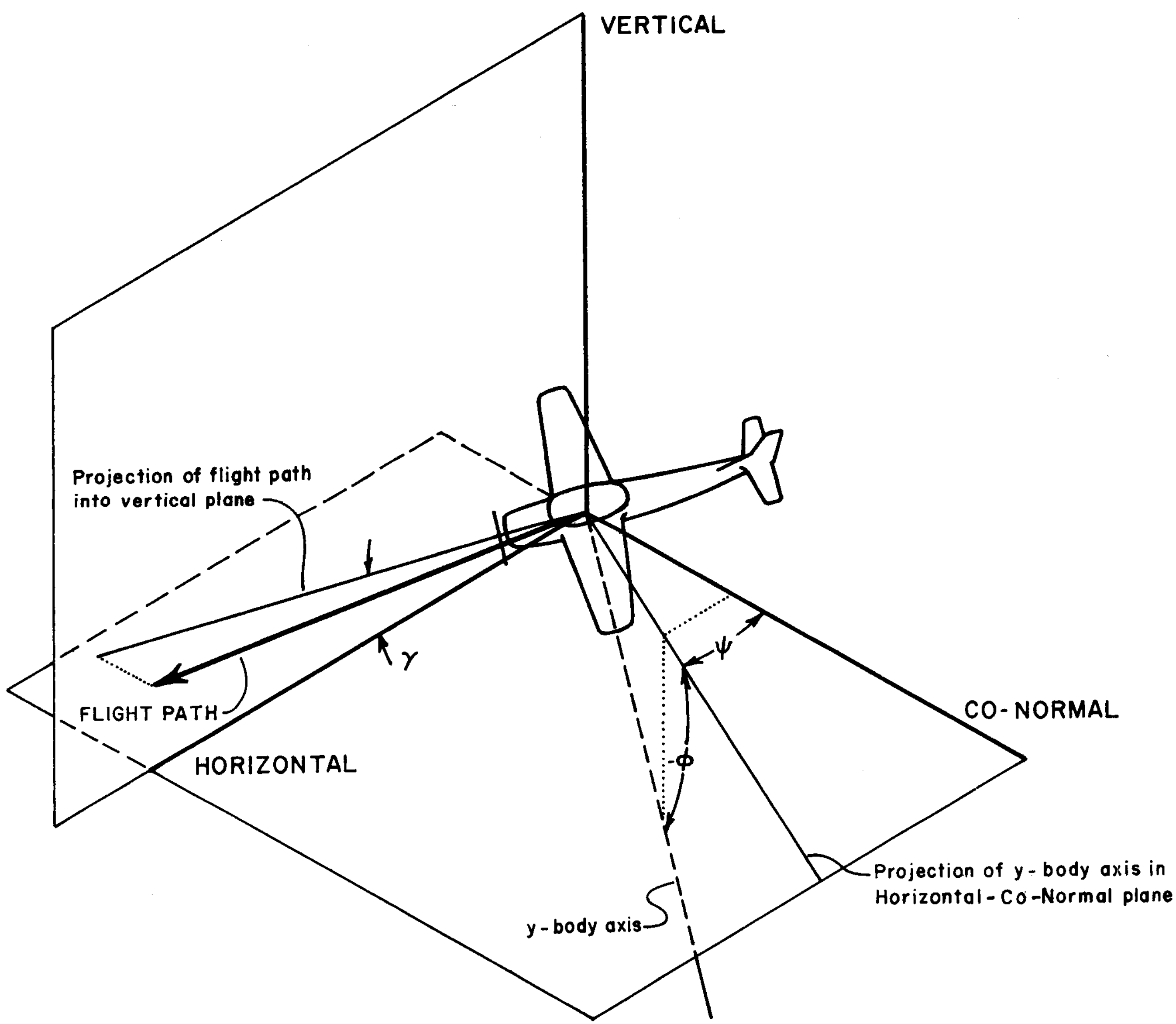

Figure B-1. Definition of flight path angles.

APPENDIX C - Point Performance Program

User Instructions

The program is written in FORTRAN IV and is designed to run in double precision on an IBM 370-165 computer with an average execution time of 1.5 seconds. This program evaluates static performance of a particular aircraft and requires the specification of the following input data:

(1) The number N of power available versus velocity data points to be specified; the control parameter ISUP which designates whether the engine is supercharged (ISUP = 1) or unsupercharged (ISUP = 0); and the reference altitude HREF (feet) which is the altitude at which the power versus velocity data points are obtained (for a supercharged aircraft HREF must be sea level in this program);

(2) The N data points of maximum power available PA (foot pounds per second) versus velocity V (feet per second) with one data point per card, power specified first;

(3) The four coefficients CDO, CDI1, CDI2, and D of the general drag polar which has the form $CD = CDO + CDI1*CL^2 + CDI2*CL^D$, and the wing area S upon which the lift and drag coefficients are based;

(4) The aircraft weight W (pounds), the initial altitude HI (feet) at which all performance calculations are made and a final altitude HF (feet). If a minimum time to climb schedule and a most economical climb schedule from HI to HF, in increments of 100 feet, are desired then HF must be greater than HI. If no schedule is desired HF must be zero.

Statements (1) through (4) represent a complete set of data for a particular aircraft. Table C-1 gives the format specification for this data.

Upon completion of the performance calculations with a given set of data the program returns to the statement where W, HI, and HF are read. In addition to specifying the aircraft weight, the variable W is used as a data input control parameter which permits the user to analyze the same aircraft for several different values of W, HI, and HF and/or analyze a completely different aircraft. The use of W as a control parameter offers the following options:

(1) If W is positive when the new values of W, HI, and HF are read the program yields a new set of performance calculations using the original drag polar and power versus velocity curve. For a given power curve and drag polar the user may exercise this option as many times as desired.

(2) If W is zero the program returns to the first read statement to obtain a complete set of data for a new aircraft. Using this

option the user may analyze as many different aircraft as he desires. Note that options (1) and (2) may be used together (see example discussed below).

(3) If W is negative the program terminates. Thus, the final data card for any computer run must have a negative value for W.

The example data set given in Table C-2 utilizes all three of the options presented above. Using this data the program calculates two sets of performance characteristics for each of the two aircraft specified. A sample output for the Cessna 182 is presented after the program listing.

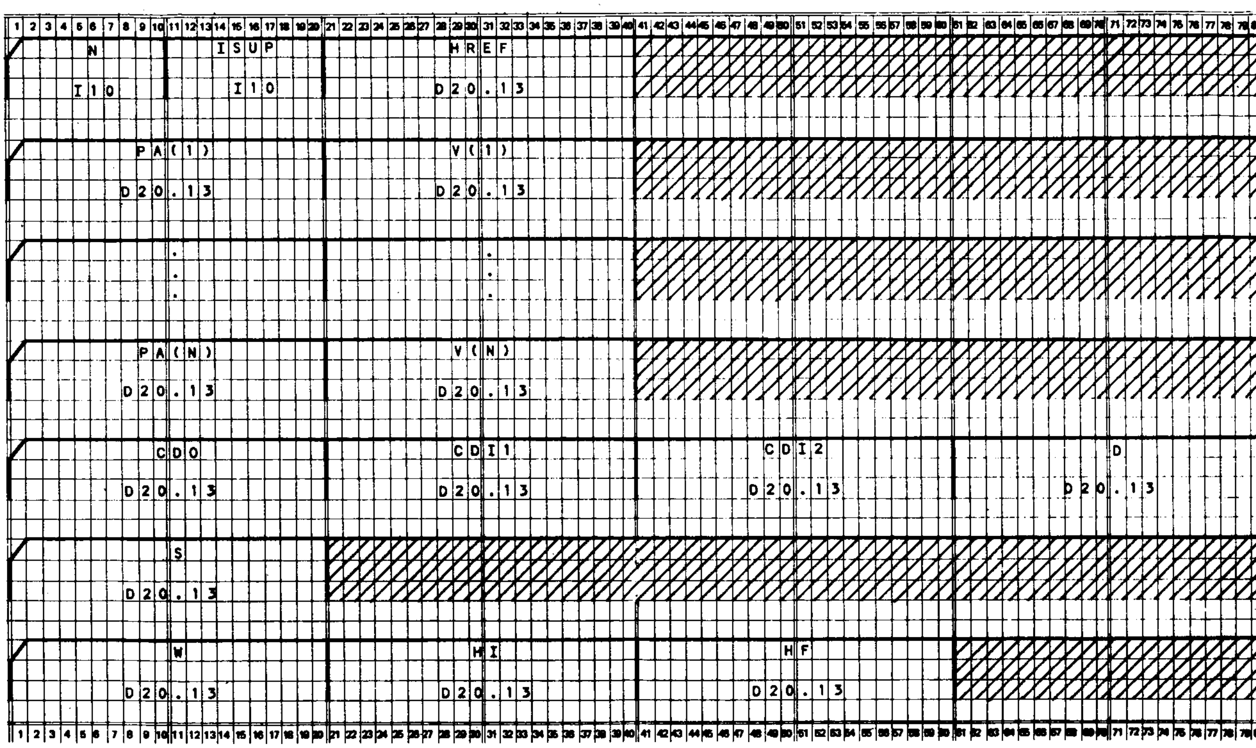

Table C-1. Format specification for input data.

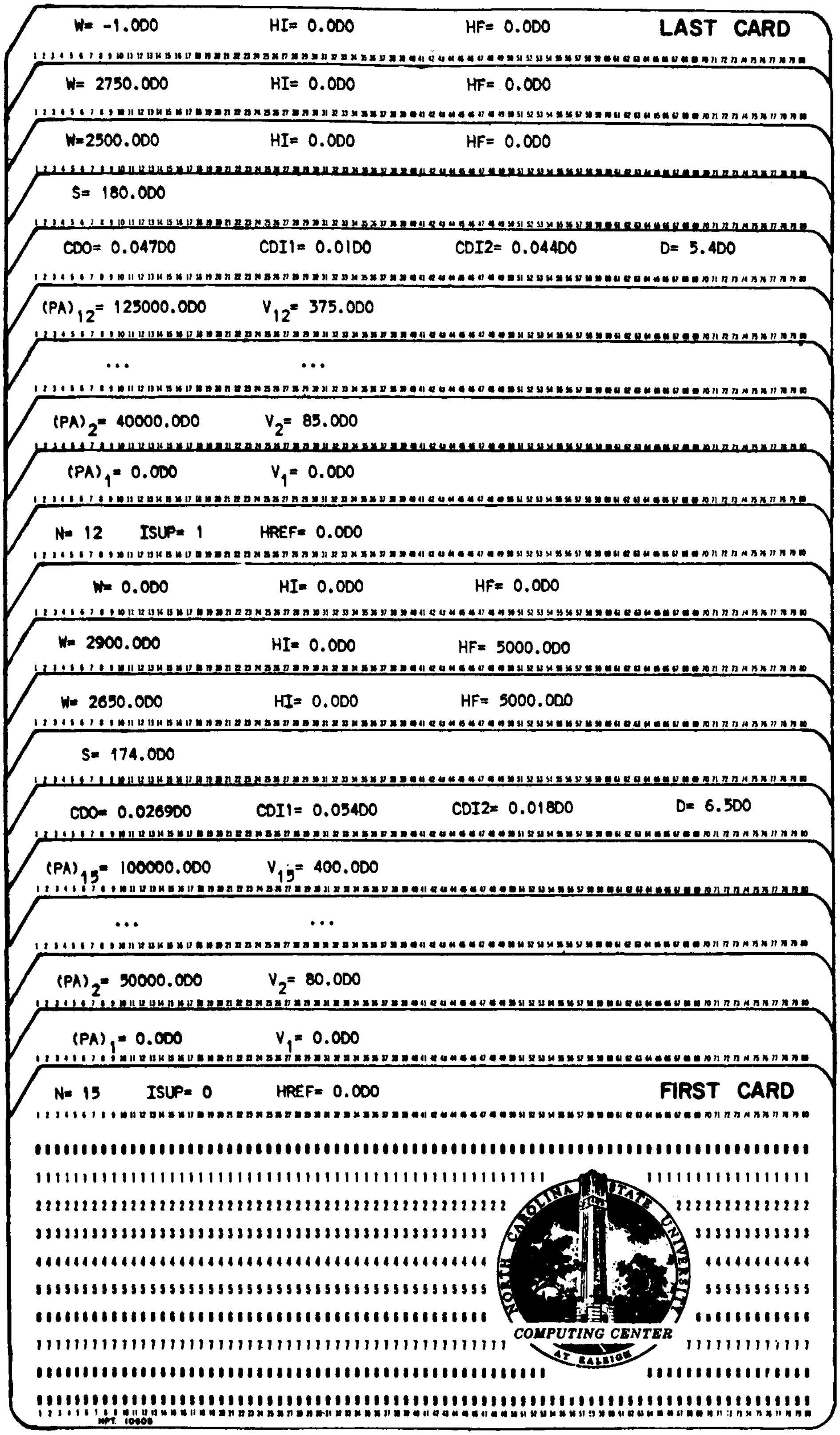

Table C-2. Example data set for point performance program.

Listing

```
C
C              **************************************
C              *                                    *
C              *                                    *
C              *      STATIC PERFORMANCE PROGRAM    *
C              *                                    *
C              *                                    *
C              **************************************
C
C
C     GIVEN VALUES OF THE AIRCRAFT CHARACTERISTICS, THE POWER
C     AVAILABLE VS. VELOCITY CURVE AT SOME REFERENCE ALTITUDE,
C     AN INITIAL ALTITUDE HI, AND A FINAL ALTITUDE HF, THIS PROGRAM
C     CALCULATES THE FOLLOWING:
C
C        1) MAXIMUM AND MINIMUM LEVEL FLIGHT SPEEDS AT ALTITUDE HI
C
C        2) SPEED FOR MAXIMUM CLIMB ANGLE AND MAXIMUM CLIMB ANGLE
C           AT ALTITUDE HI
C
C        3) SPEED FOR MINIMUM POWER (MAXIMUM ENDURANCE) AND
C           MINIMUM POWER AT ALTITUDE HI
C
C        4) CLASSICAL SPEED FOR MAXIMUM RANGE AT ALTITUDE HI
C
C        5) SERVICE AND ABSOLUTE CEILINGS (FOR NON-SUPERCHARGED
C           ENGINES)
C
C        6) MAXIMUM R/C, SPEED, POWER, AND TIME SCHEDULE VS.
C           ALTITUDE FOR MINIMUM TIME TO CLIMB FROM ALTITUDE HI
C           TO ALTITUDE HF
C
C        7) R/C, SPEED, AND POWER SCHEDULE VS. ALTITUDE FOR MOST
C           ECONOMICAL CLIMB FROM ALTITUDE HI TO ALTITUDE HF
C
C        8) MAXIMUM R/C, POWER AVAILABLE, AND POWER REQUIRED
C           SCHEDULE VS. VELOCITY AT ALTITUDE HI FOR VELOCITIES
C           BETWEEN THE MINIMUM AND MAXIMUM LEVEL FLIGHT SPEEDS
C
C        9) LIFT AND DRAG COEFFICIENTS IN ALL THE ABOVE CASES
C
C     THE DRAG POLAR IS ASSUMED TO BE OF THE FORM
C
C        CD = CD0 + CDI1*CL**2 + CDI2*CL**D
C
C     FOR A CONVENTIONAL DRAG POLAR, CDI2 AND D ARE INPUTTED AS
C     ZEROES.  IF A CONVENTIONAL DRAG POLAR IS USED, THE PROGRAM
C     WILL GIVE AN UNREALISTIC VALUE FOR THE MINIMUM LEVEL FLIGHT
C     SPEED.
C
C     THIS PROGRAM ASSUMES THAT SUPERCHARGED MEANS THAT THE POWER
C     AVAILABLE VS. VELOCITY CURVE DOES NOT VARY WITH ALTITUDE
C
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON A,AA,B,BB,C,CC,D,CD0,CDI1,CDI2,CLCOEF,COEF(13),HI,HF,HREF,
     $PA(20),PACOEF(4,20),PCR,SGMREF,SIGMA,V(20),W,ISUP,N,NM1
      DIMENSION DELV(20)
      DATA RHO/0.002380D0/
C
C
C      INPUT POWER AVAILABLE VS. VELOCITY AT REFERENCE ALTITUDE HREF
C
C        N     - NUMBER OF POINTS ON PA VS. V CURVE
C        ISUP  - CONTROL VARIABLE FOR SUPERCHARGED ENGINE
C                ISUP = 0 -> ENGINE NOT SUPERCHARGED
C                ISUP = 1 -> ENGINE SUPERCHARGED AND REFERENCE
C                            ALTITUDE MUST BE SEA LEVEL
C        HREF  - REFERENCE ALTITUDE AT WHICH POWER AVAILABLE
C                VS. VELOCITY CURVE IS GIVEN
C        PA    - POWER AVAILABLE IN FOOT-POUNDS/SECOND
C        V     - VELOCITY IN FEET/SECOND
C
C     THE FIRST POINT ON THE POWER CURVE MUST ALWAYS BE THE
C     ZERO POWER AT ZERO VELOCITY POINT, THAT IS PA(1) = 0.0
C     AND V(1) = 0.0
C
    1 READ(1,100) N,ISUP,HREF
  100 FORMAT(2I10,D20.13)
      READ(1,110) (PA(I),V(I),I=1,N)
  110 FORMAT(2D20.13)
      IF(ISUP.NE.0) GO TO 2
      WRITE(3,200) HREF
  200 FORMAT(1H1//1X,52X,'POWER AVAILABLE VS. VELOCITY'/1X,47X,'REFERENC
     $E ALTITUDE =',D12.5,1X,'FEET'//1X,50X,'PA(FT-LBS/SEC)',7X,'V(FT/SE
     $C)')
      GO TO 3
    2 WRITE(3,205) HREF
  205 FORMAT(1H1//1X,52X,'POWER AVAILABLE VS. VELOCITY'/1X,56X,'SUPERCHA
     $RGED ENGINE'/1X,47X,'REFERENCE ALTITUDE =',D12.5,' FEET'//1X,50X,'
     $PA(FT-LBS/SEC)',7X,'V(FT/SEC)')
    3 WRITE(3,210) (PA(I),V(I),I=1,N)
  210 FORMAT(1X,51X,D12.5,6X,D12.5)
C
C     OBTAIN CURVE FIT OF POWER AVAILABLE VS. VELOCITY BY THE
C     SPLINE METHOD.  THE CURVE FIT IS OF THE FORM
C
C        PA = PACOEF(1,I)*V**3 + PACOEF(2,I)*V**2
C                              + PACOEF(3,I)*V + PACOEF(4,I)
C
C     FOR              V(I) <= V <= V(I+1)
C
C     AFTER CURVE FIT IS OBTAINED, REPLACE V(1) = 0.0 WITH
C     V(1) = 1.0 AND PA(1) = 0.0 WITH THE POWER CORRESPONDING
C     TO V(1) = 1.0.  THE PURPOSE OF THIS PROCEDURE IS TO
C     PREVENT ZERO DIVIDES.
C
      NM1 = N - 1
      CALL SPLINE(N,PA,V,PACOEF)
      V(1) = 1.0D0
      PA(1) = PACOEF(1,1) + PACOEF(2,1) + PACOEF(3,1) + PACOEF(4,1)
C
C
C      INPUT AIRCRAFT DRAG POLAR
C
C        CD0   - PARASITIC DRAG COEFFICIENT
C        CDI1  - COEFFICIENT FOR FIRST INDUCED DRAG TERM IN THE
C                DRAG POLAR
C        CDI2  - COEFFICIENT FOR SECOND INDUCED DRAG TERM IN THE
C                DRAG POLAR.  IT MUST BE NON-NEGATIVE.
C                CDI2 = 0 IN A CONVENTIONAL DRAG POLAR
C        D     - EXPONENT FOR THE LIFT COEFFICIENT IN THE SECOND
C                INDUCED DRAG TERM IN THE DRAG POLAR
C                D = 0 IN A CONVENTIONAL DRAG POLAR
C        S     - WING AREA IN SQUARE FEET ON WHICH THE LIFT AND
C                DRAG COEFFICIENTS ARE BASED
C
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      READ(1,120) CDO,CDI1,CDI2,D,S
  120 FORMAT(4D20.13)
C
C
C     INPUT AIRCRAFT WEIGHT AND ALTITUDE DATA
C
C        W     - AIRCRAFT WEIGHT IN POUNDS
C                W IS ALSO A DATA INPUT CONTROL PARAMETER
C                W = 0.0 DIRECTS PROGRAM TO STATEMENT NUMBER 1
C                        TO INPUT A NEW POWER AVAILABLE VS. VE-
C                        LOCITY CURVE
C                W < 0.0 TERMINATES PROGRAM
C        HI    - INITIAL ALTITUDE AT WHICH ALL PERFORMANCE FIGURES
C                ARE CALCULATED
C        HF    - FINAL ALTITUDE FOR CALCULATIONS OF MINIMUM TIME
C                CLIMB AND MOST ECONOMICAL CLIMB FROM ALTITUDE HI
C                TO ALTITUDE HF
C                HF = 0 -> PERFORMANCE CALCULATED ONLY AT ALTITUDE
C                          HI.  NO SCHEDULES AS A FUNCTION OF
C                          ALTITUDE ARE GIVEN.
C
    4 READ(1,120) W,HI,HF
      IF(W) 10,1,5
    5 WRITE(3,220) CDO,CDI1,CDI2,D,S,W
  220 FORMAT(1X///1X,54X,'AIRCRAFT CHARACTERISTICS'//1X,32X,'CD =',D12.5
     $,' +',D12.5,'*CL**2 +',D12.5,'*CL**',D12.5/1X,36X,'WING AREA =',D1
     $2.5,' SQ.FT',6X,'WEIGHT =',D12.5,' LBS'///)
      IF(HF.EQ.0.0D0) GO TO 6
      WRITE(3,230) HI,HF
  230 FORMAT(1X,41X,'STATIC PERFORMANCE AT AN ALTITUDE =',D12.5,' FT'/1X
     $,21X,'WITH MINIMUM TIME AND MOST ECONOMICAL CLIMB SCHEDULES TO A F
     $INAL ALTITUDE =',D12.5,' FT'/1X,21X,90('+')/)
      GO TO 7
    6 WRITE(3,240) HI
  240 FORMAT(1X,41X,'STATIC PERFORMANCE AT AN ALTITUDE =',D12.5,' FT'/1X
     $,41X,50('+')/)
C
C
C     CALCULATE VARIOUS VARIABLES NEEDED BY ALL SUBROUTINES
C
C        SGMREF- RATIO OF AIR DENSITY AT REFERENCE ALTITUDE TO
C                AIR DENSITY AT SEA LEVEL
C        SIGMA - RATIO OF AIR DENSITY AT INITIAL ALTITUDE HI TO
C                AIR DENSITY AT SEA LEVEL
C        PCR   - MULTIPLICATIVE FACTOR THAT CORRECTS POWER AVAIL-
C                ABLE AT REFERENCE ALTITUDE TO POWER AVAILABLE
C                AT INITIAL ALTITUDE.  FOR SUPERCHARGED ENGINE,
C                PCR = 1.0.
C        A,AA
C        B,BB
C        C,CC  - VARIABLES REQUIRED BY ALL SUBROUTINES
C        CLCOEF- COEFFICIENT USED TO CALCULATE CL
C                CL = CLCOEF/(SIGMA*V**2)
C
    7 SGMREF = (1.0D0 - 6.86D-6*HREF)**4.26D0
      SIGMA = (1.0D0 - 6.86D-6*HI)**4.26D0
      PCR = (SIGMA - 0.165D0)/(SGMREF - 0.165D0)
      IF(ISUP.NE.0) PCR = 1.0D0
      A = -0.5D0*S*RHO*CDO
      AA = A*SIGMA
      B = -2.0D0*W*W*CDI1/(S*RHO)
      BB = B/SIGMA
      C = 0.0D0
      IF(D.NE.0.0D0) C = 2.0D0*W/S/RHO*(CDI2*W)**(1.0D0/(D-1.0D0))
      CC = C/SIGMA
      CLCOEF = 2.0D0*W/S/RHO
      CALL LEVEL(VMAX,VMIN)
      CALL ANGLE
      CALL ENDURE(VMIN)
      CALL RANGE
      CALL CEILNG
      CALL CLIMB
      CALL ECONOM
C
C     GENERATE MAXIMUM R/C, POWER AVAILABLE, AND POWER REQUIRED
C     SCHEDULE VS. VELOCITY
C
      HWORK = HI
      WRITE(3,250) HI
  250 FORMAT(1X,37X,'MAXIMUM R/C, POWER AVAILABLE, & POWER REQUIRED VS V
     $ELOCITY'/1X,57X,'AT ',D12.5,' FT'//1X,34X,'R/C(FT/SEC)',5X,'PA(FT-
     $LBS/SEC)',4X,'PRQ(FT-LBS/SEC)',6X,'V(FT/SEC)')
      PAV = POWER(VMIN,HWORK)
      PREQ = -AA*VMIN**3 - BB/VMIN
      IF(D.NE.0.0D0) PREQ = PREQ + VMIN*(CC/VMIN**2)**(D-1.0D0)
      RC = (PAV - PREQ)/W
      WRITE(3,260) RC,PAV,PREQ,VMIN
  260 FORMAT(1X,33X,4(D12.5,6X))
      ISPEED = 0.1D0*VMIN
      ISPEED = 10*ISPEED + 10
    8 SPEED = ISPEED
      IF(SPEED.GE.VMAX) GO TO 9
      PAV = POWER(SPEED,HWORK)
      PREQ = -AA*SPEED**3 - BB/SPEED
      IF(D.NE.0.0D0) PREQ = PREQ + SPEED*(CC/SPEED**2)**(D-1.0D0)
      RC = (PAV - PREQ)/W
      WRITE(3,260) RC,PAV,PREQ,SPEED
      ISPEED = ISPEED + 10
      GO TO 8
    9 PAV = POWER(VMAX,HWORK)
      PREQ = -AA*VMAX**3 - BB/VMAX
      IF(D.NE.0.0D0) PREQ = PREQ + VMAX*(CC/VMAX**2)**(D-1.0D0)
      RC = (PAV - PREQ)/W
      WRITE(3,260) RC,PAV,PREQ,VMAX
      WRITE(3,270)
  270 FORMAT(1X///)
C
C     RETURN TO STATEMENT NUMBER 4 FOR NEW DATA
C
      GO TO 4
   10 CALL EXIT
      END
```

```
C
C
C     THIS SUBROUTINE CALCULATES THE MAXIMUM AND MINIMUM LEVEL
C     FLIGHT SPEEDS
C
C
C        VMAX  - MAXIMUM LEVEL FLIGHT SPEED
C                IF PROGRAM FAILS TO FIND A MAXIMUM LEVEL FLIGHT
C                SPEED, IT SETS VMAX = V(N) OF THE PA VS. V CURVE
C        VMIN  - MINIMUM LEVEL FLIGHT SPEED
C                IF PROGRAM FAILS TO FIND A MINIMUM LEVEL FLIGHT
C                SPEED, IT SETS VMIN = V(1) OF THE PA VS. V CURVE
C        COEF  - COEFFICIENTS OF THE PSUEDO-POLYNOMIAL (SEE FUNC-
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```
C                 TION POLY FCR DEFINITION OF PSUEDO-POLYNOMIAL)
C                 WHOSE RCOTS ARE VMAX ANC VMIN
C          KOUNT - COUNTER FCR THE NUMBER CF ROOTS FOUND
C
      SUBROUTINE LEVEL(VMAX,VMIN)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON A,AA,B,BB,C,CC,D,CD0,CDI1,CDI2,CLCOEF,COEF(13),HI,HF,HREF,
     $PA(20),PACOEF(4,20),PCR,SGMREF,SIGMA,V(20),W,ISUP,N,NM1
      DIMENSION NCIM(5)
      DATA NDIM/4,0,2,2,2/
C
C     DETERMINE ROOTS OF PSUEDO-PCLYNOMIAL BETWEEN V(1) AND V(N)
C
      IRCOT = 0
      KOUNT = 0
      COEF(5) = BB
      COEF(6) = D
      COEF(7) = CC
      COEF(8) = 1.0D0
      I = 1
    1 COEF(1) = PACOEF(1,I)*PCR + AA
      COEF(2) = PACOEF(2,I)*PCR
      COEF(3) = PACOEF(3,I)*PCR
      COEF(4) = PACOEF(4,I)*PCR
      V1 = V(I)
      V2 = V(I+1)
      CALL ROOT(NDIM,COEF,V1,V2,VROOT,IRCOT)
      IF(IROOT) 4,4,2
    2 IF(KOUNT-1) 3,5,5
C
C     FIRST ROOT FOUND.  CONTINUE SEARCH FOR SECOND ROOT.
C
    3 VMIN = VROOT
      KOUNT = 1
      IF(IROOT.EQ.2) I = I + 1
      IROOT = C
    4 I = I + 1
      IF(I.LT.N) GO TO 1
      IF(KOUNT) 7,7,8
C
C     TWO ROOTS FOUND
C
    5 VMAX = VROOT
    6 CL = CLCOEF/(SIGMA*VMIN*VMIN)
      CD = CD0 + CDI1*CL**2 + CDI2*CL**D
      WRITE(3,200) VMIN,CL,CC
  200 FORMAT(1X,42X,'MINIMUM LEVEL FLIGHT SPEED =',D12.5,' FT/SEC'/1X,34
     $X,'LIFT COEFFICIENT =',D12.5,4X,'DRAG CCEFFICIENT =',D12.5/)
      CL = CLCOEF/(SIGMA*VMAX*VMAX)
      CD = CD0 + CDI1*CL**2 + CDI2*CL**D
      WRITE(3,210) VMAX,CL,CD
  210 FORMAT(1X,42X,'MAXIMUM LEVEL FLIGHT SPEED =',D12.5,' FT/SEC'/1X,34
     $X,'LIFT COEFFICIENT =',D12.5,4X,'DRAG CCEFFICIENT =',D12.5////)
      RETURN
C
C     NO ROOTS FOUNC.  SET VMAX = V(N) AND VMIN = V(1).
C
    7 VMIN = V(1)
      VMAX = V(NM1+1)
      WRITE(3,220)
  220 FORMAT(1X,1C('*'),' PRCGRAM HAS FAILED TC FIND A MAXIMUM AND A MIN
     $IMUM LEVEL FLIGHT SPEED'/1X,11X,'VMAX IS SET EQUAL TC V(N) AND VMI
     $N IS SET EQUAL TO V(1) OF THE PA(I) VS V(I) CURVE'/)
      GO TO 6
C
C     ONLY ONE ROOT HAS BEEN FOUND.  DETERMINE IF IT IS VMIN OR VMAX.
C
    8 VTEST = C.95CC*VMIN
      HTEST = HI
      TEST = PCWER(VTEST,HTEST) + AA*VTEST**3 + BB/VTEST
      IF(CC.NE.0.0C0) TEST = TEST - VTEST*(CC/VTEST**2)**(D-1.0D0)
      IF(TEST.LT.0.0D0) GO TO 9
C
C     SINGLE RCOT IS VMAX.  SET VMIN = V(1).
C
      VMAX = VMIN
      VMIN = V(1)
      WRITE(3,230)
  230 FORMAT(1X,10('*'),' PROGRAM HAS FAILED TO FIND A MINIMUM LEVEL FLI
     $GHT SPEED'/1X,11X,'VMIN IS SET EQUAL TC V(1) OF THE PA(I) VS V(I)
     $CURVE'/)
      GO TO 6
C
C     SINGLE ROOT IS VMIN.  SET VMAX = V(N).
C
    9 VMAX = V(NM1+1)
      WRITE(3,24C)
  240 FORMAT(1X,10('*'),' PRCGRAM HAS FAILED TO FIND A MAXIMUM LEVEL FLI
     $GHT SPEED'/1X,11X,'VMAX IS SET EQUAL TC V(N) OF THE PA(I) VS V(I)
     $CURVE'/)
      GO TO 6
      END
```

```
C
C
C     THIS SUBROUTINE CALCULATES THE VELCCITY FOR MAXIMUM CLIMB
C     ANGLE AND THE MAXIMUM CLIMB ANGLE
C
C
C        VANGLE - VELOCITY FCR MAXIMUM CLIMB ANGLE
C        GAMMAX - MAXIMUM CLIMB ANGLE
C        COEF   - COEFFICIENTS OF THE PSUEDO-POLYNOMIAL WHOSE ROOT
C                 IS THE VELOCITY FOR MAXIMUM CLIMB ANGLE
C
      SUBROUTINE ANGLE
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON A,AA,B,BB,C,CC,D,CD0,CDI1,CDI2,CLCOEF,COEF(13),HI,HF,HREF,
     $PA(20),PACOEF(4,20),PCR,SGMREF,SIGMA,V(20),W,ISUP,N,NM1
      DIMENSION NCIM(5)
      DATA NDIM/4,0,2,2,2/
C
C     DETERMINE ROOTS OF PSUEDO-PCLYNOMIAL BETWEEN V(1) AND V(N)
C     AND CHOOSE THE ONE GIVING THE MAXIMUM CLIMB ANGLE
      IROOT = 0
      VANGLE = C.0D0
      GAMMAX = 0.0D0
      COEF(3) = 0.0C0
      COEF(5) = -2.0D0*BB
      COEF(6) = D
      COEF(7) = CC
      COEF(8) = 2.0D0*(1.0D0-D)
      I = 1
    1 COEF(1) = 2.0C0*(AA + PACOEF(1,I)*PCR)
      COEF(2) = PACOEF(2,I)*PCR
      COEF(4) = -PACOEF(4,I)*PCR
      V1 = V(I)
```

```
      V2 = V(I+1)
      CALL ROOT(NDIM,COEF,V1,V2,VROOT,IROOT)
      IF(IROOT) 4,4,2
C
C     ROOT OF PSUEDO-POLYNOMIAL HAS BEEN FOUND.  CALCULATE CLIMB
C     ANGLE AND COMPARE IT TO CLIMB ANGLE OF ANY PREVIOUS ROOT TO
C     DETERMINE WHICH IS THE GREATER.
C
    2 GAMMA = (0.5D0*COEF(1)*VROOT*VROOT + COEF(2)*VROOT + PACOEF(3,I)*
     $PCR - COEF(4)/VROOT - 0.5D0*COEF(5)/VROOT/VROOT)/W
      IF(D.NE.0.0D0) GAMMA = GAMMA -(CC/VROOT**2)**(D-1.0D0)/W
      IF(IROOT.EQ.2) I = I + 1
      IF(GAMMA.LT.GAMMAX) GO TO 3
      GAMMAX = GAMMA
      VANGLE = VROOT
    3 IROOT = 0
    4 I = I + 1
      IF(I.LT.N) GO TO 1
      IF(VANGLE.EQ.0.0D0) GO TO 5
C
C     PRINT RESULTS
C
      GAMMAX = GAMMAX*180.0D0/3.14159D0
      CL = CLCOEF/(SIGMA*VANGLE*VANGLE)
      CD = CD0 + CDI1*CL**2 + CDI2*CL**D
      WRITE(3,200) GAMMAX,VANGLE,CL,CD
  200 FORMAT(1X,47X,'MAXIMUM CLIMB ANGLE =',D12.5,' DEG'/1X,39X,'VELOCIT
     $Y FOR MAXIMUM CLIMB ANGLE =',D12.5,' FT/SEC'/1X,34X,'LIFT COEFFICI
     $ENT =',D12.5,4X,'DRAG COEFFICIENT =',D12.5////)
      RETURN
C
C     NO ROOT HAS BEEN FOUND BETWEEN V(1) AND V(N).  WRITE ERROR
C     MESSAGE.
C
    5 WRITE(3,210)
  210 FORMAT(1X,10('*'),' PROGRAM HAS FAILED TO FIND A MAXIMUM CLIMB ANG
     $LE'////)
      RETURN
      END
```

```
C
C
C     THIS SUBROUTINE CALCULATES THE VELOCITY AND POWER FOR
C     MAXIMUM ENDURANCE
C
C
C         VENDMX - VELOCITY FOR MAXIMUM ENDURANCE
C         PENDMX - POWER FOR MAXIMUM ENDURANCE
C         COEF   - COEFFICIENTS OF PSUEDO-POLYNOMIAL WHOSE ROOT IS
C                  THE VELOCITY FOR MAXIMUM ENDURANCE
C
      SUBROUTINE ENDURE(VMIN)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON A,AA,B,BB,C,CC,D,CD0,CDI1,CDI2,CLCOEF,COEF(13),HI,HF,HREF,
     $PA(20),PACOEF(4,20),PCR,SGMREF,SIGMA,V(20),W,ISUP,N,NP1
      DIMENSION NDIM(5)
      DATA NDIM/4,0,2,2,2/
C
C     TEST TO SEE IF DRAG POLAR IS CONVENTIONAL TYPE
C
      IF(CC.NE.0.0D0) GO TO 1
C
C     OBTAIN VENDMX FROM EXACT EXPRESSION FOR THE CASE OF A
C     CONVENTIONAL DRAG POLAR
C
      VENDMX = DSQRT(DSQRT(BB/AA/3.0D0))
      PENDMX = -AA*VENDMX*VENDMX*VENDMX - BB/VENDMX
      CL = CLCOEF/(SIGMA*VENDMX*VENDMX)
      CD = CD0 + CDI1*CL**2 + CDI2*CL**D
      WRITE(3,200) VENDMX,PENDMX,CL,CD
  200 FORMAT(1X,40X,'VELOCITY FOR MAXIMUM ENDURANCE =',D12.5,' FT/SEC'/1
     $X,40X,'POWER FOR MAXIMUM ENDURANCE =',D12.5,' FT-LBS/SEC'/1X,34X,'
     $LIFT COEFFICIENT =',D12.5,4X,'DRAG COEFFICIENT =',D12.5////)
      RETURN
C
C     SOLVE A PSUEDO-POLYNOMIAL FOR VENDMX FOR THE CASE OF A
C     NON-CONVENTIONAL DRAG POLAR
C
    1 IROOT = 0
      COEF(1) = -3.0D0*AA
      COEF(2) = 0.0D0
      COEF(3) = 0.0D0
      COEF(4) = 0.0D0
      COEF(5) = BB
      COEF(6) = D
      COEF(7) = CC
      COEF(8) = 2.0D0*D - 3.0D0
      II = 1
    2 I = N - II
      V1 = V(I)
      V2 = V(I+1)
      CALL ROOT(NDIM,COEF,V1,V2,VROOT,IROOT)
      IF(IROOT.NE.0) GO TO 3
      II = II + 1
      IF(II.LT.N) GO TO 2
C
C     NO ROOT HAS BEEN FOUND BETWEEN V(1) AND V(N).  WRITE ERROR
C     MESSAGE.
C
      WRITE(3,210)
  210 FORMAT(1X,10('*'),' PROGRAM HAS FAILED TO FIND A VELOCITY FOR MAXI
     $MUM ENDURANCE'////)
      RETURN
C
C     CHECK TO SEE IF VENDMX IS LESS THAN VMIN. IF SO, SET
C     VENDMX = VMIN.
C
    3 IF(VROOT.LT.VMIN) VROOT = VMIN
C
C     CALCULATE PENDMX FROM VENDMX AND WRITE VENDMX AND PENDMX
C
      PENDMX = -AA*VROOT*VROOT*VROOT - BB/VROOT + VROOT*(CC/VROOT**2)**(
     $D-1.0D0)
      CL = CLCOEF/(SIGMA*VROOT*VROOT)
      CD = CD0 + CDI1*CL**2 + CDI2*CL**D
      WRITE(3,200) VROOT,PENDMX,CL,CD
      RETURN
      END
```

```
C
C
C     THIS SUBROUTINE CALCULATES THE CLASSICAL VELOCITY FOR
C     MAXIMUM RANGE
C
```

```
C
C        VRNGMX - VELOCITY FOR MAXIMUM RANGE
C        COEF   - COEFFICIENTS OF PSUEDO-POLYNOMIAL WHOSE ROOT
C                 IS THE VELCCITY FOR MAXIMUM RANGE
C
      SUBROUTINE RANGE
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON A,AA,B,BB,C,CC,C,CDC,CDI1,CDI2,CLCOEF,CCEF(13),HI,HF,HREF,
     $PA(20),PACOEF(4,2C),PCR,SGMREF,SIGMA,V(2C),W,ISUP,N,NM1
      DIMENSION NDIM(5)
      DATA NDIM/4,0,2,2,2/
C
C     TEST TO SEE IF CRAG POLAR IS CONVENTIONAL TYPE
C
      IF(CC.NE.0.0D0) GO TO 1
C
C     OBTAIN VRNGMX FROM EXACT EXPRESSION FOR THE CASE OF A
C     CONVENTIONAL CRAG POLAR
C
      VRNGMX = DSQRT(DSQRT(BB/AA))
      CL = CLCOEF/(SIGMA*VRNGMX*VRNGMX)
      CD = CDO + CDI1*CL**2 + CDI2*CL**D
      WRITE(3,200) VRNGMX,CL,CD
  200 FORMAT(1X,37X,'VELOCITY FOR CLASSICAL MAXIMUM RANGE =',D12.5,' FT/
     $SEC'/1X,34X,'LIFT COEFFICIENT =',D12.5,4X,'DRAG COEFFICIENT =',D12
     $.5////)
      RETURN
C
C     SOLVE A PSUECO-POLYNOMIAL FOR VRNGMX FCR THE CASE OF A
C     NON-CONVENTIONAL DRAG POLAR
C
    1 IROOT = 0
      COEF(1) = -AA
      COEF(2) = 0.0C0
      CCEF(3) = 0.0D0
      COEF(4) = C.0D0
      COEF(5) = BB
      COEF(6) = D
      CCEF(7) = CC
      COEF(8) = D - 1.0D0
      I = 1
    2 V1 = V(I)
      V2 = V(I+1)
      CALL ROOT(NDIM,COEF,V1,V2,VROOT,IROOT)
      IF(IROOT.NE.0) GO TO 3
      I = I + 1
      IF(I.LT.N) GO TO 2
C
C     NO ROOT FOUND BETWEEN V(1) AND V(N).  WRITE ERROR
C     MESSAGE.
C
      WRITE(3,210)
  210 FORMAT(1X,10('*'),' PROGRAM HAS FAILED TO FIND A VELOCITY FOR MAXI
     $MUM CLASSICAL RANGE'////)
      RETURN
C
C     ROOT FOUND
C
    3 CL = CLCOEF/(SIGMA*VROOT*VROOT)
      CD = CDO + CDI1*CL**2 + CDI2*CL**D
      WRITE(3,200) VROOT,CL,CD
      RETURN
      END
```

```
C
C
C     THIS SUBROUTINE CALCULATES THE SERVICE CEILING AND THE
C     ABSOLUTE CEILING FOR THE CASE OF A NON-SUPERCHARGED ENGINE
C
C
C        VSERVC - VELOCITY AT SERVICE CEILING
C        VABS   - VELOCITY AT ABSOLUTE CEILING
C        HSERVC - SERVICE CEILING
C        HABS   - ABSCLUTE CEILING
C        COEF   - COEFFICIENTS OF EITHER OF TWO PSUEDO-POLYNOMIALS.
C                 ROOT OF FIRST POLYNCMIAL IS VELCCITY AT EITHER
C                 SERVICE OR ABSOLUTE CEILING.  ROOT OF SECOND
C                 POLYNOMIAL IS EITHER THE SERVICE OR ABSOLUTE
C                 CEILING.  THE PROGRAM ITERATES BETWEEN THE TWO
C                 POLYNOMIALS UNTIL CONVERGENCE IS OBTAINED ON
C                 BOTH THE VELOCITY AND THE ALTITUDE.
C        ERROR  - CONVERGENCE CRITERIA FOR BOTH VELOCITY
C                 AND ALTITUDE
C        DHDT   - RATE OF CLIMB AT EITHER SERVICE CR ABSOLUTE
C                 CEILING.  VALUE OF THIS PARAMETER CETERMINES
C                 WHETHER PROGRAM IS SOLVING FOR SERVICE CEILING
C                 OR ABSOLUTE CEILING.
C                 DHDT = 1.67 FOR SERVICE CEILING SOLUTION
C                 DHDT = 0.00 FOR ABSOLUTE CEILING SOLUTION
C        KOUNT  - COUNTER FOR NUMBER OF ITERATIONS ON BOTH
C                 VELOCITY AND ALTITUDE.  IF NUMBER OF ITERATIONS
C                 EXCEEDS 100, ITERATION CEASES AND PROGRAM USES
C                 THE 100TH ITERATED VALUES OF VELOCITY AND
C                 ALTITUDE.
C        ITRY   - CONTROL VARIABLE FOR FAILURE TO FINC A ROOT OF
C                 EITHER VELOCITY OR ALTITUDE POLYNOMIAL.  PROGRAM
C                 USES HREF AS ITS INITIAL GUESS FOR THE SERVICE
C                 CEILING.  IF THERE IS A FAILURE TO FINC A ROOT
C                 OF EITHER POLYNOMIAL, THE PROGRAM CHANGES ITRY
C                 FROM 0 TO 1 AND BEGINS AGAIN WITH 1.0D4 AS A NEW
C                 GUESS AT THE SERVICE CEILING.  IF FAILURE OCCURS
C                 AGAIN, THE PROGRAM TERMINATES WITH AN ERROR
C                 MESSAGE, RESETS ITRY TO 0, AND BEGINS THE ITERA-
C                 TION FOR THE ABSOLUTE CEILING USING THE LAST
C                 ITERATED VALUE FOR THE SERVICE CEILING AS ITS
C                 INITIAL GUESS FOR THE ABSOLUTE CEILING.  IF
C                 THERE IS A FAILURE TO FIND A ROOT OF EITHER
C                 POLYNOMIAL, PROGRAM REPEATS PROCEDURE USED FOR
C                 SERVICE CEILING.  UPON TERMINATION DUE TO SECOND
C                 FAILURE, PROGRAM RETURNS TO MAIN LINE.
C
      SUBROUTINE CEILNG
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON A,AA,B,BB,C,CC,D,CDO,CDI1,CDI2,CLCOEF,COEF(13),HI,HF,HREF,
     $PA(20),PACOEF(4,20),PCR,SGMREF,SIGMA,V(20),W,ISUP,N,NM1
      DIMENSION NDIMV(5),NDIMSG(5)
      DATA NDIMV/4,0,2,2,2/,NDIMSG/2,C,1,1,1/,ERRCR/1.0D-04/
C
C
C     TEST TO DETERMINE IF ENGINE IS SUPERCHARGED
C
      IF(ISUP.NE.0) GO TO 14
C
C     SET UP PARAMETERS TO BEGIN ITERATICN FOR SERVICE CEILING
C
      DHDT = 1.6666666666666667D0
```

```
      TRMSGM = SGMREF - 0.165D0
      SGMWRK = SGMREF
      PCRWRK = 1.0D0
      HWORK = HREF
      VOLD = 1.0D0
      ITRY = 0
      KOUNT = 0
C
C     OBTAIN SOLUTION OF VELOCITY POLYNOMIAL
C
    1 IROOT = 0
      CCEF(4) = 0.0D0
      COEF(5) = -B/SGMWRK
      CCEF(6) = D
      COEF(7) = C/SGMWRK
      COEF(8) = 3.0D0 - 2.0D0*D
      II = 1
    2 I = N - II
      COEF(1) = 3.0D0*(PACOEF(1,I)*PCRWRK + A*SGMWRK)
      COEF(2) = 2.0D0*PACOEF(2,I)*PCRWRK
      COEF(3) = PACOEF(3,I)*PCRWRK
      V1 = V(I)
      V2 = V(I+1)
      CALL ROOT(NDIMV,COEF,V1,V2,VROOT,IROOT)
      IF(IROOT) 3,3,4
    3 II = II + 1
      IF(II.LT.N) GO TO 2
      GO TO 6
C
C     OBTAIN SOLUTION OF ALTITUDE POLYNOMIAL
C
    4 P = POWER(VROOT,HREF)
      COEF(1) = P*VROOT/TRMSGM + A*VROOT**4
      COEF(2) = -0.165D0*P*VROOT/TRMSGM - W*DHDT*VROOT
      CCEF(3) = B
      CCEF(4) = D
      COEF(5) = C/VROOT**2
      COEF(6) = VROOT**2
      IRCCT = 0
      SGM2 = 1.0D0
      I = 1
    5 SGM1 = SGM2
      SGM2 = SGM1 - 0.0125D0
      CALL ROOT(NDIMSG,COEF,SGM1,SGM2,SGROOT,IROOT)
      IF(IROOT.NE.0) GO TO 9
      I = I + 1
      IF(I.LT.79) GO TO 5
C
C     FAILURE TO FIND ROOT OF EITHER POLYNOMIAL HAS OCCURED.  TEST
C     TO SEE IF THIS IS SECOND FAILURE.
C
    6 IF(ITRY.EQ.1) GO TO 7
C
C     FIRST FAILURE.  RE-INITIALIZE PARAMETERS WITH NEW GUESS FOR
C     CEILING ALTITUDE.
C
      HWCRK = 1.0D04
      PCRWRK = (SGMWRK - 0.165D0)/TRMSGM
      SGMWRK = (1.0D0 - 6.86D-6*HWORK)**4.26D0
      ITRY = 1
      GO TO 1
C
C     SECOND FAILURE.  WRITE APPROPRIATE ERROR MESSAGE.
C
    7 IF(DHDT.EQ.0.0D0) GO TO 8
      WRITE(3,200)
  200 FORMAT(1X,10('*'),' PROGRAM HAS FAILED TO FIND SERVICE CEILING'/)
      GO TO 11
    8 WRITE(3,210)
  210 FORMAT(1X,10('*'),' PROGRAM HAS FAILED TO FIND THE ABSOLUTE CEILIN
     $G'////)
      GO TO 13
C
C     TEST FOR CONVERGENCE
C
    9 SGMOLD = SGMWRK
      SGMWRK = SGROOT
      KOUNT = KOUNT + 1
      IF(DABS(1.0D0-VROOT/VOLD).LT.ERROR.AND.DABS(1.0D0-SGMWRK/SGMOLD).L
     $T.ERROR) GO TO 10
      VOLD = VROOT
      PCRWRK = (SGMWRK - 0.165D0)/TRMSGM
      IF(KOUNT.LT.100) GO TO 1
      WRITE(3,220)
  220 FORMAT(1X,10('*'),' NUMBER OF ITERATIONS IN SUBROUTINE CEILING EXC
     $EEDS 100'/1X,11X,'PROGRAM SETS VSERVC & HSERVC OR VABS & HABS TO 1
     $00TH ITERATED VALUES'/)
   10 IF(DHDT.EQ.0.0D0) GO TO 12
C
C     WRITE SERVICE CEILING RESULTS
C
      VSERVC = VROOT
      HSERVC = (1.0D0 - SGMWRK**(1.0D0/4.26D0))*1.0D6/6.86D0
      CL = CLCOEF/(SGMWRK*VSERVC*VSERVC)
      CD = CD0 + CDI1*CL**2 + CDI2*CL**D
      WRITE(3,230) HSERVC,VSERVC,CL,CD
  230 FORMAT(1X,50X,'SERVICE CEILING =',D12.5,' FT'/1X,44X,'VELOCITY AT
     $SERVICE CEILING =',D12.5,' FT/SEC'/1X,34X,'LIFT COEFFICIENT =',D12
     $.5,4X,'DRAG COEFFICIENT =',D12.5/)
C
C     INITIALIZE PARAMETERS FOR ITERATION FOR ABSOLUTE CEILING
C
   11 ITRY = 0
      DHDT = 0.0D0
      KOUNT = 0
      VOLD = 1.0D0
      GO TO 1
C
C     WRITE ABSOLUTE CEILING RESULTS
C
   12 VABS = VROOT
      HABS = (1.0D0 - SGMWRK**(1.0D0/4.26D0))*1.0D6/6.86D0
      CL = CLCOEF/(SGMWRK*VABS*VABS)
      CD = CD0 + CDI1*CL**2 + CDI2*CL**D
      WRITE(3,240) HABS,VABS,CL,CD
  240 FORMAT(1X,49X,'ABSOLUTE CEILING =',D12.5,' FT'/1X,41X,'VELOCITY AT
     $ ABSOLUTE CEILING =',D12.5,' FT/SEC'/1X,34X,'LIFT COEFFICIENT =',D
     $12.5,4X,'DRAG COEFFICIENT =',D12.5////)
   13 RETURN
C
C     WRITE MESSAGE FOR CASE OF SUPERCHARGED ENGINE
C
   14 WRITE(3,250)
  250 FORMAT(1X,31X,'SERVICE AND ABSOLUTE CEILINGS NOT CALCULATED FOR SU
     $PERCHARGED ENGINE'////)
      RETURN
      END
```

```
C
C
C     THIS SUBROUTINE CALCULATES THE VELOCITY FOR MAXIMUM RATE OF
C     CLIMB AND MAXIMUM RATE OF CLIMB.
C
C     IF A NON-ZERO FINAL ALTITUDE HF IS GIVEN,  THIS SUBROUTINE
C     CALCULATES A MAXIMUM R/C,SPEED,POWER, AND TIME SCHEDULE VS.
C     ALTITUDE FOR MINIMUM TIME CLIMB FROM ALTITUDE HI TO ALTITUDE
C     HF.
C
C
C        VRCMAX - VELOCITY FOR MAXIMUM RATE OF CLIMB
C        RCMAX  - MAXIMUM RATE OF CLIMB
C        PRCMAX - POWER FOR MAXIMUM RATE OF CLIMB
C        TIME   - INTEGRATED CLIMB TIME FROM INITIAL ALTITUDE HI
C                 TO INTERMEDIATE ALTITUDE HIWORK.  INTEGRATION
C                 IS BY THE TRAPEZOIDAL RULE.
C        HIWORK - INTERMEDIATE ALTITUDE VARIABLE WHICH IS INCRE-
C                 MENTED FROM ALTITUDE HI TO ALTITUDE HF IN STEPS
C                 OF DELTAH.  USER MAY SET DELTAH TO ANY VALUE
C                 DESIRED.  PRESENT VALUE OF DELTAH IS SET AT 50.
C        COEF   - COEFFICIENTS OF PSUEDO-POLYNOMIAL WHOSE ROOT
C                 IS THE VELOCITY FOR MAXIMUM RATE OF CLIMB
C
      SUBROUTINE CLIMB
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON A,AA,B,BB,C,CC,D,CDO,CDI1,CDI2,CLCOEF,COEF(13),HI,HF,HREF,
     $PA(20),PACOEF(4,20),PCR,SGMREF,SIGMA,V(20),W,ISUP,N,NM1
      DIMENSION NDIM(5)
      DATA NDIM/4,0,2,2,2/
C
C     DETERMINE ROOTS OF PSUEDO-POLYNOMIAL BETWEEN V(1) AND V(N)
C     AND CHOOSE THE ONE GIVING THE MAXIMUM RATE OF CLIMB
C
      TIME = 0.0D0
      DELTAH = 500.0D0
      HIWORK = HI
      SGMWRK = SIGMA
      PCRWRK = PCR
      COEF(4) = 0.0D0
      IF(HF.EQ.0.0D0) GO TO 1
      WRITE(3,200) HI,HF
  200 FORMAT(1X,32X,'MAXIMUM RATE OF CLIMB SCHEDULE FROM',D12.5,' FT TO'
     $,D12.5,' FT'//1X,10X,'H(FT)',10X,'R/C(FT/SEC)',8X,'V(FT/SEC)',7X,'
     $P(FT-LBS/SEC)',10X,'CL',16X,'CD',14X,'T(SEC)')
    1 IROOT = 0
      RCMAX = 0.0D0
      VRCMAX = 0.0D0
      COEF(5) = -B/SGMWRK
      COEF(6) = D
      COEF(7) = C/SGMWRK
      COEF(8) = 3.0D0 - 2.0D0*D
      I = 1
    2 COEF(1) = 3.0D0*(PACOEF(1,I)*PCRWRK + A*SGMWRK)
      COEF(2) = 2.0D0*PACOEF(2,I)*PCRWRK
      COEF(3) = PACOEF(3,I)*PCRWRK
      V1 = V(I)
      V2 = V(I+1)
      CALL ROOT(NDIM,COEF,V1,V2,VROOT,IROOT)
      IF(IROOT) 4,4,3
C
C     ROOT OF PSUEDO-POLYNOMIAL HAS BEEN FOUND.  CALCULATE RATE OF
C     CLIMB AND COMPARE IT TO RATE OF CLIMB OF ANY PREVIOUS ROOT
C     TO DETERMINE WHICH IS THE GREATER.
C
    3 RC =(((COEF(1)*VROOT/3.0D0 + 0.5D0*COEF(2))*VROOT + COEF(3))*VROOT
     $+ PACOEF(4,I)*PCRWRK - COEF(5)/VROOT)/W
      IF(D.NE.0.0D0) RC = RC -VROOT*(COEF(7)/VROOT**2)**(D-1.0D0)/W
      IF(IROOT.EQ.2) I = I + 1
      IROOT = 0
      IF(RC.LT.RCMAX) GO TO 4
      RCMAX = RC
      VRCMAX = VROOT
    4 I = I + 1
      IF(I.LT.N) GO TO 2
      IF(VRCMAX.NE.0.0D0) GO TO 5
C
C     NO ROOT HAS BEEN FOUND BETWEEN V(1) AND V(N).  WRITE ERROR
C     MESSAGE.
C
      WRITE(3,210)
  210 FORMAT(1X,10('*'),' PROGRAM HAS FAILED TO FIND A MAXIMUM RATE OF C
     $LIMB'////)
      RETURN
    5 PRCMAX = POWER(VRCMAX,HIWORK)
      CL = CLCOEF/(SGMWRK*VRCMAX*VRCMAX)
      CD = CDO + CDI1*CL**2 + CDI2*CL**D
      IF(HF.NE.0.0D0) GO TO 6
C
C     NO MINIMUM TIME CLIMB SCHEDULE GENERATED.  WRITE RCMAX,VRCMAX,
C     AND PRCMAX FOR ALTITUDE HI.
C
      WRITE(3,220) RCMAX,VRCMAX,PRCMAX,CL,CD
  220 FORMAT(1X,45X,'MAXIMUM RATE OF CLIMB =',D12.5,' FT/SEC'/1X,38X,'VE
     $LOCITY FOR MAXIMUM RATE OF CLIMB =',D12.5,' FT/SEC'/1X,38X,'POWER
     $FOR MAXIMUM RATE OF CLIMB =',D12.5,' FT-LBS/SEC'/1X,34X,'LIFT COEF
     $FICIENT =',D12.5,4X,'DRAG COEFFICIENT =',D12.5////)
      RETURN
C
C     CALCULATE INTEGRATED CLIMB TIME BY TRAPEZOIDAL RULE
C
    6 IF(HIWORK.NE.HI) TIME = TIME + 0.5D0*DELTAH*(1.0D0/RCHLD + 1.0D0/R
     $CMAX)
      WRITE(3,230) HIWORK,RCMAX,VRCMAX,PRCMAX,CL,CD,TIME
  230 FORMAT(1X,6X,7(D12.5,6X))
      RCHLD = RCMAX
C
C     TEST TO SEE IF FINAL ALTITUDE HF HAS BEEN REACHED
C
      IF(HIWORK.EQ.HF) GO TO 8
      HIWORK = HIWORK + DELTAH
      IF((HF-HIWORK).GT.1.0D0) GO TO 7
      DELTAH = HF - (HIWORK - DELTAH)
      HIWORK = HF
    7 SGMWRK = (1.0D0 - 6.86D-6*HIWORK)**4.26D0
      IF(ISUP.EQ.0) PCRWRK = (SGMWRK - 0.165D0)/(SGMREF - 0.165D0)
      GO TO 1
    8 WRITE(3,240)
  240 FORMAT(1X///)
      RETURN
      END
```

```
C     THIS SUBROUTINE CALCULATES THE VELOCITY FOR THE MOST
C     ECONOMICAL RATE OF CLIMB AND THE MOST ECONOMICAL RATE OF
```

```
C     CLIMB.
C
C     IF A NON-ZERO FINAL ALTITUDE HF IS GIVEN, THIS SUBROUTINE
C     CALCULATES A R/C, SPEED, AND POWER SCHEDULE VS. ALTITUDE FOR
C     MOST ECONOMICAL CLIMB FROM ALTITUDE HI TC ALTITUDE HF.
C
C
C        VECONM - VELOCITY FOR MOST ECONOMICAL RATE OF CLIMB
C        RCECNM - MOST ECONCMICAL RATE OF CLIMB
C        PECONM - POWER FOR MOST ECONOMICAL RATE OF CLIMB
C        HIWORK - INTERMEDIATE ALTITUDE VARIABLE WHICH IS INCRE-
C                 MENTED FROM ALTITUDE HI TO ALTITUDE HF IN STEPS
C                 OF DELTAH.  USER MAY SET DELTAH TO ANY VALUE
C                 DESIRED.  PRESENT VALUE OF DELTAH IS SET AT 50.
C        COEF   - COEFFICIENTS OF PSUEDO-POLYNOMIAL WHOSE ROOT
C                 IS THE VELOCITY FOR MOST ECONOMICAL RATE OF CLIMB
C
      SUBROUTINE ECONOM
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON A,AA,B,BB,C,CC,D,CD0,CDI1,CDI2,CLCOEF,COEF(13),HI,HF,HREF,
     $PA(20),PACOEF(4,20),PCR,SGMREF,SIGMA,V(20),W,ISUP,N,NM1
      DIMENSION NDIM(5)
      DATA NDIM/4,2,0,3,2/
C
C     DETERMINE ROOT OF PSUEDO-POLYNOMIAL BETWEEN V(1) AND V(N)
C
      DELTAH = 500.0D0
      HIWORK = HI
      SGMWRK = SIGMA
      PCRWRK = PCR
      IF(HF.EQ.0.0D0) GO TO 1
      WRITE(3,200) HI,HF
  200 FORMAT(1X,28X,'MOST ECONOMICAL RATE OF CLIMB SCHEDULE FROM',D12.5,
     $' FT TO',D12.5,' FT'//1X,19X,'H(FT)',10X,'R/C(FT/SEC)',8X,'V(FT/SE
     $C)',7X,'P(FT-LBS/SEC)',10X,'CL',16X,'CD')
    1 IROOT = 0
      AAWRK = A*SGMWRK*PCRWRK
      BBWRK = B*PCRWRK/SGMWRK
      COEF(8) = D
      COEF(9) = C/SGMWRK
      II = 1
    2 I = N - II
      COEF(1) = PACOEF(2,I)*AAWRK
      COEF(2) = 2.0D0*PACOEF(3,I)*AAWRK
      COEF(3) = 3.0DC*PACOEF(4,I)*AAWRK
      COEF(4) = -4.0D0*PACOEF(1,I)*BBWRK
      COEF(5) = -3.0D0*PACOEF(2,I)*BBWRK
      COEF(6) = -2.CD0*PACOEF(3,I)*BBWRK
      COEF(7) = -PACOEF(4,I)*BBWRK
      COEF(10) = (3.0D0-2.0D0*D)*PCRWRK*PACOEF(4,I)
      COEF(11) = 2.0D0*(1.0D0-D)*PCRWRK*PACOEF(3,I)
      COEF(12) = (1.0D0-2.0D0*D)*PCRWRK*PACOEF(2,I)
      COEF(13) = -2.0D0*D*PCRWRK*PACOEF(1,I)
      V1 = V(I)
      V2 = V(I+1)
      CALL ROOT(NDIM,COEF,V1,V2,VROOT,IROOT)
      IF(IROOT) 3,3,4
    3 II = II + 1
      IF(II.LT.N) GO TO 2
C
C     NO ROOT FOUND BETWEEN V(1) AND V(N).  WRITE ERRCR
C     MESSAGE.
C
      WRITE(3,210)
  210 FORMAT(1X,10('+'),' PROGRAM HAS FAILED TO FIND A MOST ECONOMICAL R
     $ATE OF CLIMB'////)
      RETURN
    4 VECONM = VROOT
      PECONM = POWER(VECONM,HIWORK)
      RCECNM = (PECONM + (AAWRK*VECONM**3 + BBWRK/VECONM)/PCRWRK)/W
      IF(D.NE.0.0D0) RCECNM = RCECNM - VECONM*(COEF(9)/VECONM**2)**(D-1.
     $0D0)/PCRWRK/W
      CL = CLCOEF/(SGMWRK*VECONM*VECONM)
      CD = CD0 + CDI1*CL**2 + CDI2*CL**D
      IF(HF.NE.0.0D0) GO TO 5
C
C     NO MOST ECONOMICAL CLIMB SCHEDULE GENERATED.  WRITE RCECNM,
C     VECONM, AND PECONM FOR ALTITUDE HI.
C
      WRITE(3,220) RCECNM,VECONM,PECONM,CL,CD
  220 FORMAT(1X,41X,'MOST ECCNOMICAL RATE OF CLIMB =',D12.5,' FT/SEC'/1X
     $,38X,'VELOCITY FOR MOST ECONOMICAL CLIMB =',D12.5,' FT/SEC'/1X,38X
     $,'POWER FOR MOST ECONOMICAL CLIMB =',D12.5,' FT-LBS/SEC'/1X,34X,'L
     $IFT COEFFICIENT =',D12.5,4X,'DRAG COEFFICIENT =',D12.5////)
      RETURN
    5 WRITE(3,230) HIWORK,RCECNM,VECONM,PECONM,CL,CD
  230 FORMAT(1X,15X,6(D12.5,6X))
C
C     TEST TO SEE IF FINAL ALTITUDE HF HAS BEEN REACHED.
C
      IF(HIWORK.EQ.HF) GO TO 6
      HIWORK = HIWORK + DELTAH
      IF((HF-HIWORK).LT.1.0D0) HIWORK = HF
      SGMWRK = (1.0D0 - 6.88D-6*HIWORK)**4.26D0
      IF(ISUP.EQ.0) PCRWRK = (SGMWRK - 0.165D0)/(SGMREF - 0.165D0)
      GO TO 1
    6 WRITE(3,240)
  240 FORMAT(1X///)
      RETURN
      END
```

```
C
C
C     THIS FUNCTION CALCULATES THE POWER AVAILABLE AT A GIVEN
C     VELOCITY AND ALTITUDE USING THE CURVE FIT OF PA VS. V GEN-
C     ERATED BY THE SPLINE METHOD.  THE FORM OF THE CURVE FIT IS
C
C        PA = PACOEF(1,I)*V**3 + PACOEF(2,I)*V**2
C                              + PACOEF(3,I)*V + PACOEF(4,I)
C
C     FOR               V(I) <= V <= V(I+1).
C
C     THE CURVE FIT GIVES THE POWER AVAIABLE AT THE REFERENCE
C     ALTITUDE.  THIS POWER IS THEN CORRECTED TO THE GIVEN
C     ALTITUDE.
C
C
C        VV     - VELOCITY AT WHICH POWER AVAILABLE IS DESIRED
C        HH     - ALTITUDE AT WHICH POWER AVAILABLE IS DESIRED
C        POWER  - RETURNED VALUE OF POWER AVAILABLE
C
      FUNCTION POWER(VV,HH)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON A,AA,B,BB,C,CC,D,CDC,CDI1,CDI2,CLCOEF,COEF(13),HI,HF,HREF,
     $PA(20),PACOEF(4,20),PCR,SGMREF,SIGMA,V(2C),W,ISUP,N,NM1
C
```

```
C     IF VV IS LESS THAN V(1).  EXTRAPOLATE TO VV USING PACOEF'S
C     THAT ARE FOR VELOCITY RANGE V(1) TO V(2).
C
      IF(VV.GT.V(1)) GO TO 2
      I = 1
      GO TO 4
C
C     DETERMINE POSITION OF VV WITH RESPECT TC VELOCITY RANGE
C     V(1) TC V(N)
C
    2 DO 3 I=1,NM1
      IF(V(I).LE.VV.AND.V(I+1).GT.VV) GO TO 4
    3 CONTINUE
C
C     VV IS GREATER THAN V(N).  EXTRAPOLATE TO VV USING PACOEF'S
C     THAT ARE FOR VELOCITY RANGE V(N-1) TO V(N).
C
      I = NM1
C
C     CALCULATE POWER AVAILABLE AT VELOCITY VV AT REFERENCE
C     ALTITUDE
C
    4 A1 = PACOEF(1,I)
      A2 = PACOEF(2,I)
      A3 = PACOEF(3,I)
      A4 = PACOEF(4,I)
      POWER = ((A1*VV + A2)*VV + A3)*VV + A4
C
C     CORRECT POWER AVAILABLE TO GIVEN ALTITUDE HH
C     IF ENGINE IS SUPERCHARGED, HREF IS ASSUMED TO BE SEA LEVEL
C     ANC SEA LEVEL POWER IS RETURNED
C
      IF(ISUP.NE.0) RETURN
      SGMWRK = (1.0D0 - 6.86D-6*HH)**4.26D0
      POWER = POWER*(SGMWRK - C.165D0)/(SGMREF - 0.165D0)
      RETURN
      END
```

```
C
C
C     THIS FUNCTION EVALUATES A PSUEDO-POLYNOMIAL.  A PSUEDO-
C     POLYNOMIAL HAS THE FORM
C
C        P(X) = A(1)*X**N1 + A(2)*X**(N1-1) + ... +A(N1)*X +
C               A(N1+1) + A(N1+2)/X + ... + A(N1+1+N2)/X**N2
C                       - (B(1)*X**N3 + B(2)*X**(N3+1) + ...
C                       + B(N4-N3+1)*X**N4)*(C/X**N5)**(D-1)
C
C
C        X      - ARGUEMENT OF POLYNOMIAL
C        POLY   - VALUE OF PCLYNOMIAL RETURNED
C        N      - FIVE ELEMENT ARRAY CONTAINING THE FIVE DEGREE
C                 NUMBERS CF THE PSUEDO-POLYNOMIAL
C                 N(1) = N1;  N(2) = N2;  N(3) = N3
C                             N(4) = N4; N(5) = N5
C        A      - (N1+N2-N3+N4+4) ELEMENT ARRAY CONTAINING THE
C                 PSUEDO-POLYNOMIAL CCEFFICIENTS
C                 A(1) TO A(N1+N2+1) - CORRESPOND TO A-COEFFICIENTS
C                 A(N1+N2+2) - CORRESPONDS TO EXPONENT D
C                 A(N1+N2+3) - CORRESPONDS TO C-COEFFICIENT
C                 A(N1+N2+4) TO A(N1+N2-N3+N4+4) - CORRESPONDS TO
C                                                  B-COEFFICIENTS
C
      FUNCTION POLY(N,A,X)
      REAL*8 A(13),POLY,POLY2,X
      DIMENSION N(5)
C
C     CALCULATE CONVENTIONAL PORTION OF PSUEDO-POLYNOMIAL
C
      M = N(1) + 1
      POLY = A(1)
      DO 1 I=2,M
    1 POLY = POLY*X + A(I)
      IF(N(2).EQ.0) GO TO 3
      MM = N(2)
      DO 2 I=1,MM
    2 POLY = POLY + A(M+I)/X**I
C
C     CALCULATE UNCONVENTIONAL PORTION OF PSUEDO-POLYNOMIAL
C     IF EXPONENT D = 0, UNCONVENTIONAL PART IS NOT CALCULATED
C
    3 M = M + N(2) + 1
      IF(A(M).EQ.0.0D0) RETURN
      L = N(3) + 1
      LL = N(4) + 1
      PCLY2 = 0.0D0
      DO 4 II=L,LL
      I = II - 1
      J = II - L + 1
    4 POLY2 = POLY2 + A(M+1+J)*X**I
      POLY = POLY - POLY2*(A(M+1)/X**N(5))**(A(M) - 1.0D0)
      RETURN
      END
```

```
C
C
C     THIS SUBROUTINE DETERMINES IF A ROOT OF A PSUEDO-POLYNOMIAL
C     LIES BETWEEN ARGUEMENTS X1 AND X2 AND EVALUATES THE ROOT
C
C
C        NDIM   - THREE ELEMENT ARRAY CONTAINING THE THREE DEGREE
C                 NUMBERS OF THE PSUEDO-POLYNOMIAL
C        COEF   - ARRAY CONTAINING THE COEFFICIENTS OF THE PSUEDO-
C                 POLYNOMIAL
C        X1,X2  - TWO ARGUEMENTS BETWEEN WHICH A ROOT IS SOUGHT
C        XROOT  - ROOT OF THE PSUEDO-POLYNOMIAL
C        IROOT  - CONTROL VARIABLE THAT SIGNIFIES IF AND WHERE
C                 A ROOT IS FOUND
C                 IROOT = 0 -> NO ROOT FOUND BETWEEN X1 AND X2.
C                              IROOT MUST BE INITIALIZED TO ZERO
C                              IN THE CALLING PROGRAM AND MUST BE
C                              RE-INITIALIZED IN THE CALLING PRO-
C                              GRAM AFTER A ROOT HAS BEEN FOUND
C                              BEFORE THE NEXT CALL TO THIS SUB-
C                              ROUTINE.
C                 IROOT = 1 -> THE ROOT IS EITHER X1 OR BETWEEN X1
C                              AND X2
C                 IROOT = 2 -> THE ROOT IS X2
C        ERROR  - RELATIVE CONVERGENCE CRITERIA ON XROOT.  USER
C                 MAY SET ITS VALUE AS DESIRED.  PRESENT VALUE IS
C                 1.0D-04.
C
      SUBROUTINE ROOT(NDIM,COEF,X1,X2,XROOT,IROOT)
      IMPLICIT REAL*8(A-H,O-Z)
```

```
      DIMENSION COEF(13),NDIM(5)
      DATA ERROR/1.0D-04/
C
C     DETERMINE IF THE ROOT IS X1
C
      F1 = POLY(NDIM,COEF,X1)
      IF(F1.NE.0.0D0) GO TO 1
      XROOT = X1
      IROOT = 1
      RETURN
C
C     DETERMINE IF ROOT IS X2 OR BETWEEN X1 AND X2
C
    1 F2 = POLY(NDIM,COEF,X2)
      IF(F1*F2) 3,9,2
C
C     NO ROOT
C
    2 RETURN
C
C     ROOT IS BETWEEN X1 AND X2.  SOLVE FOR ROOT USING METHOD OF
C     FALSE POSITION.
C
    3 XHOLD = X1
      XOLD = 1.0D0
      DO 7 J=1,200
      XROOT = (X1*F2 - X2*F1)/(F2 - F1)
      FROOT = POLY(NDIM,COEF,XROOT)
      IF(F1*FROOT) 4,8,5
    4 X2 = XROOT
      F2 = FROOT
      GO TO 6
    5 X1 = XROOT
      F1 = FROOT
    6 IF(DABS(1.0D0-XROOT/XOLD).LT.ERROR.AND.DABS(FROOT).LT.1.0D-04) GO
     $TO 8
      XOLD = XROOT
    7 CONTINUE
      WRITE(3,200)
  200 FORMAT(1X,10('*'),' ITERATION ON XROOT FAILS TO CONVERGE IN 200 IT
     $ERATIONS'/1X,11X,'SUBROUTINE ROOT RETURNS THE 200TH ITERATED VALUE
     $ OF XROOT')
    8 X1 = XHOLD
      IROOT = 1
      RETURN
C
C     THE ROOT IS X2
C
    9 XROOT = X2
      IROOT = 2
      RETURN
      END
```

```
C
C
C     THIS SUBROUTINE PERFORMS A SPLINE FIT ON THE TABULATED
C     DATA Y VS. X.  THE SPLINE FIT PROVIDES A CURVE FIT OF THE
C     TABULATED DATA THAT HAS A CONTINUOUS FIRST DERIVATIVE.  THE
C     FORM OF THE CURVE FIT IS
C
C        Y = AA(1,I)*X**3 + AA(2,I)*X**2
C                       + AA(3,I)*X + AA(4,I)
C
C     FOR        X(I) <= X <= X(I+1).
C
C
C        N     - NUMBER OF TABULATED DATA POINTS
C        Y     - TABULATED FUNCTION VALUES
C        X     - TABULATED ARGUEMENT VALUES
C        AA    - ARRAY OF COEFFICIENTS OF THE CUBIC POLYNOMIAL
C                SPLINE FIT.  DIMENSIONS OF AA ARE 4 BY N-1.
C
C     REFERENCE FOR THIS METHOD IS
C
C        THE THEORY OF SPLINES AND THEIR APPLICATIONS
C        J.H. AHLBERG, ET AL, ACADEMIC PRESS, NEW YORK , 1967
C
      SUBROUTINE SPLINE(N,Y,X,AA)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION AA(4,20),H(20),Q(20),U(20),X(20),Y(20)
C
      NM1 = N - 1
      DO 1 I=1,NM1
    1 H(I) = X(I+1) - X(I)
C
C
C     MODIFIED LEFT HAND END CONDITION THAT ALEVIATES THE NEED
C     TO SPECIFY THE X-DERIVATIVE OF Y AT POINT 1
C
      Q(1) = -31.0D0/32.0D0
      H1 = H(1)
      H2 = H(2)
      H3 = H(3)
      U(1) = Y(1)*(32.0D0*H1 + 42.0D0*H2 + 21.0D0*H3)/(H1+H2)/(H1+H2+H3)
     $- Y(2)*(11.0D0*H1 + 42.0D0*H2 + 21.0D0*H3)/(H2+H3)/H2 + Y(3)*H1*(
     $11.0D0*H1 + 21.0D0*(H2 + H3))/(H1+H2)/H2/H3 - Y(4)*H1*(11.0D0*H1 +
     $21.0D0*H2)/(H2+H3)/(H1+H2+H3)/H3
C
C     GENERATE INTERNAL U(I) BY ALGORITHM GIVEN BY AHLBERG
C
      U(1) = 3.0D0*U(1)/H1/16.0D0
      HM = H(1)
      YY = Y(2)
      YM = Y(1)
      DO 2 I=2,NM1
      HH = H(I)
      YP = Y(I+1)
      D = 3.0D0*((YP - YY)/HH - (YY - YM)/HM)/(HH + HM)
      C = 0.5D0*HH/(HH + HM)
      A = 0.5D0 - C
      P = A*Q(I-1) + 1.0D0
      Q(I) = -C/P
      U(I) = (D - A*U(I-1))/P
      HM = HH
      YM = YY
    2 YY = YP
C
C     MODIFIED RIGHT HAND END CONDITION THAT ALEVIATES THE NEED
C     TO SPECIFY THE X-DERIVATIVE OF Y AT POINT N
C
      A = 31.0D0/32.0D0
      P = A*Q(N-1) + 1.0D0
      H1 = H(NM1)
      H2 = H(NM1-1)
      H3 = H(NM1-2)
      C = Y(N)*(32.0D0*H1 + 42.0D0*H2 + 21.0D0*H3)/(H1+H2)/(H1+H2+H3) -
```

```
     $Y(NM1)*(11.0D0*H1 + 42.0D0*H2 + 21.0D0*H3)/(H2+H3)/H2 + Y(NM1-1)*
     $H1*(11.0D0*H1 + 21.0D0*(H2 + H3))/(H1+H2)/H2/H3 - Y(NM1-2)*H1*(11.
     $0D0*H1 + 21.0D0*H2)/(H2+H3)/(H1+H2+H3)/H3
      D = 3.0D0*D/H1/16.0D0
      U(N) = (D - A*U(N-1))/P
C
C     SOLVE FOR THE SPLINE COEFFICIENTS CORRESPONDING TO AHLBERG'S
C     M(0) TO M(N) AND STORE THEM IN THE U(I).
C
      DO 3 J=1,NM1
      I = N - J
    3 U(I) = Q(I)*U(I+1) + U(I)
C
C     FORM THE AA(J,I) COEFFICIENTS FOR THE CCNVENTIONAL FCRM OF
C    A  CUBIC POLYNOMIAL FROM THE U(I)
C
      UU = U(1)
      XX = X(1)
      YY = Y(1)
      DO 4 I=1,NM1
      UP = U(I+1)
      XP = X(I+1)
      YP = Y(I+1)
      HH = H(I)
      AA(1,I) = (UP - UU)/HH/6.0D0
      AA(2,I) = 0.5D0*(XP*UU - XX*UP)/HH
      AA(3,I) = 0.5D0*(UP*XX*XX - UU*XP*XP)/HH + (UU - UP)*HH/6.0D0 +
     $(YP - YY)/HH
      AA(4,I) = (UU*XP*XP*XP - UP*XX*XX*XX)/HH/6.0D0 + (UP*XX - UU*XP)*H
     $H/6.0D0 + (YY*XP - YP*XX)/HH
      XX = XP
      UU = UP
    4 YY = YP
      RETURN
      END
```

Sample Output

POWER AVAILABLE VS. VELOCITY
REFERENCE ALTITUDE = 0.0 FEET

PA(FT-LBS/SEC)	V(FT/SEC)
0.0	0.0
0.291500 05	0.273300 02
0.524700 05	0.546700 02
0.699600 05	0.820000 02
0.816200 05	0.109330 03
0.874500 05	0.136670 03
0.909480 05	0.16400C 03
0.944460 05	0.191330 03
0.958650 05	0.218670 03
0.967780 05	0.246000 03
0.973610 05	0.273330 03
0.979440 05	0.300670 03
0.9947C0 05	0.328000 03
0.9947C0 05	0.355330 03
0.994700 05	0.382660 03

AIRCRAFT CHARACTERISTICS

CD = 0.268800-01 • 0.542420-01*CL**2 + 0.177510-01*CL** 0.650000 01
WING AREA = 0.174C0D 03 SQ.FT   WEIGHT = 0.26500D 04 LBS

STATIC PERFORMANCE AT AN ALTITUDE = 0.0 FT
WITH MINIMUM TIME AND MOST ECONOMICAL CLIMB SCHEDULES TO A FINAL ALTITUDE = 0.100000 05 FT
++

MINIMUM LEVEL FLIGHT SPEED = 0.904650 02 FT/SEC
LIFT COEFFICIENT = C.156380 01   DRAG COEFFICIENT = 0.484180 00

MAXIMUM LEVEL FLIGHT SPEED = 0.252570 03 FT/SEC
LIFT COEFFICIENT = 0.20062D 00   DRAG COEFFICIENT = 0.290630-01

MAXIMUM CLIMB ANGLE = 0.102200 02 DEG
VELOCITY FOR MAXIMUM CLIMB ANGLE = 0.11710C 03 FT/SEC
LIFT COEFFICIENT = C.933380 00   DRAG COEFFICIENT = 0.854750-01

VELOCITY FOR MAXIMUM ENDURANCE = 0.125710 03 FT/SEC
POWER FOR MAXIMUM ENDURANCE = 0.275450 05 FT-LBS/SEC
LIFT COEFFICIENT = 0.809850 00   DRAG COEFFICIENT = 0.66961C-01

VELOCITY FOR CLASSICAL MAXIMUM RANGE = 0.142050 03 FT/SEC
LIFT COEFFICIENT = C.634230 00   DRAG COEFFICIENT = 0.496190-01

SERVICE CEILING = 0.194420 05 FT
VELOCITY AT SERVICE CEILING = 0.175540 03 FT/SEC
LIFT COEFFICIENT = 0.764240 00   DRAG COEFFICIENT = ^.616530-01

ABSOLUTE CEILING = 0.212360 05 FT
VELOCITY AT ABSOLUTE CEILING = 0.180230 03 FT/SEC
LIFT COEFFICIENT = C.770560 00   DRAG COEFFICIENT = 0.623480-01

MAXIMUM RATE OF CLIMB SCHEDULE FROM 0.0 FT TC 0.10000D 05 FT

H(FT)	R/C(FT/SEC)	V(FT/SEC)	P(FT-LBS/SEC)	CL	CD	T(SEC)
0.0	0.222570 02	0.136010 03	0.873540 05	0.691830 00	0.544600-01	0.0
0.500000 03	0.216590 02	0.136620 03	0.859210 05	0.695810 00	0.548210-01	0.227750 02
0.100000 04	0.210650 02	0.137250 03	0.845040 05	0.699660 00	0.551740-01	0.461850 02
0.150000 04	0.204750 02	0.137890 03	0.831020 05	0.703390 00	0.555190-01	0.702630 02
0.200000 04	0.198890 02	0.138560 03	0.817160 05	0.707000 00	0.558560-01	0.950420 02
0.250000 04	0.193070 02	0.139250 03	0.803450 05	0.710490 00	0.561850-01	0.120560 03
0.300000 04	0.187280 02	0.139960 03	0.789890 05	0.713840 00	0.565040-01	0.146860 03
0.350000 04	0.18154C 02	0.140690 03	0.776470 05	0.717080 00	0.568140-01	0.173980 03
0.400000 04	0.175830 02	0.141450 03	0.763200 C5	0.720180 00	0.571150-01	0.201970 03
0.450000 04	0.170150 02	0.142220 03	0.75C080 C5	0.723160 00	0.574050-01	0.230880 03
0.500000 04	0.164510 02	0.143020 03	0.737090 05	C.726000 00	0.576840-01	0.260770 03
0.550000 04	0.158910 02	0.143840 03	0.724250 05	0.728720 00	0.579530-01	0.291700 03
0.600000 04	0.153350 02	0.144680 03	0.711550 05	0.731310 00	0.582110-01	0.323730 C3
0.650000 04	0.147820 02	0.145540 C3	0.698990 05	0.733760 00	0.584570-01	0.356950 C3
0.700000 04	0.142330 02	0.146430 03	0.686570 05	0.736080 00	0.586910-01	0.391420 03
0.750000 04	0.136870 02	0.147340 03	0.674280 05	0.738280 00	0.589140-01	0.427260 03
0.800000 04	0.131440 02	0.148280 03	0.662130 05	0.740340 00	0.591240-01	0.464540 03
0.850000 04	0.126060 02	0.149240 03	0.650110 05	0.742270 00	0.593220-01	0.503390 03
0.900000 04	0.120700 02	0.150220 03	0.638230 05	0.744070 00	0.595080-01	0.543940 03
0.950000 04	0.115390 02	0.151220 03	0.626490 05	0.745730 00	0.596810-01	0.586320 03
0.100000 05	0.110100 02	0.152260 03	0.614880 05	0.747270 00	0.598410-01	0.630690 03

MOST ECONOMICAL RATE OF CLIMB SCHEDULE FROM 0.0 FT TO 0.100000 05 FT

H(FT)	R/C(FT/SEC)	V(FT/SEC)	P(FT-LBS/SEC)	CL	CD
0.0	0.221090 02	0.12897D 03	0.86228D 05	0.76940D 00	0.62220D-01
0.500000 03	0.21514D 02	0.12986D 03	C.84877D 05	0.77016D 00	0.62304D-01
0.10000D 04	0.20922D 02	0.13075D 03	0.83536D 05	0.77089D 00	0.62385D-01
0.15000D 04	0.20332D 02	0.13166D 03	0.82205D 05	0.77160D 00	0.62464D-01
0.20000D 04	0.19745D 02	0.13258D 03	0.80885D 05	0.77227C 00	0.62539D-01
0.25000D 04	0.19160D 02	0.13351D 03	0.79575D 05	0.77291D 00	0.62610D-01
0.30000D 04	0.18577D 02	0.13445D 03	0.78275D 05	0.77352D 00	0.62678D-01
0.35000D 04	0.17997D 02	0.13541D 03	0.76986D 05	0.77409D 00	0.62743D-01
0.40000D 04	0.17420D 02	0.13639D 03	0.75708D 05	0.77463D 00	0.62803D-01
0.45000D 04	0.16845D 02	0.13737D 03	0.74440D 05	0.77512D 00	0.62858D-01
0.50000D 04	0.16272D 02	0.13837D 03	0.73184D 05	0.77559C 00	0.62911D-01
0.55000D 04	C.15702D 02	0.13938D 03	0.71938D 05	0.77603D 00	0.62960D-01
0.60000D 04	0.15135C 02	0.14041D 03	0.70703D 05	0.77643D 00	0.63005D-01
0.65000D 04	0.14570D 02	0.14145D 03	0.69480D 05	0.77679C 00	0.63047D-01
0.70000D 04	C.14007D 02	0.14251D 03	0.68267D 05	0.77712D 00	0.63084D-01
0.75000D 04	0.13447D 02	0.14359D 03	0.67066D 05	0.77741C 00	0.63116D-01
0.80000D 04	0.12890D 02	0.14468D 03	0.65876D 05	0.77765D 00	0.63144D-01
0.85000D 04	C.12335D 02	0.14578D 03	0.64697D 05	0.77785D 00	0.63166D-01
0.90000D 04	0.11782D 02	0.14691D 03	0.63530D 05	0.77799D 00	0.63183D-01
0.95000D 04	0.11232D 02	0.148C5D 03	0.62375D 05	0.77809D 00	0.63193D-01
0.10000D 05	0.10684C 02	0.14921D 03	0.61231D 05	0.77813C 00	0.63198D-01

MAXIMUM R/C, POWER AVAILABLE, & POWER REQUIRED VS VELOCITY
AT 0.0 FT

R/C(FT/SEC)	PA(FT-LBS/SEC)	PRQ(FT-LBS/SEC)	V(FT/SEC)
0.39105D-09	0.74225D 05	0.74225D 05	0.90465D 02
0.13626D 02	0.78343D 05	0.42234D 05	0.10000C 03
0.19113D 02	0.81825D 05	0.31176D 05	0.11000D 03
0.21340D 02	0.84449D 05	0.27899D 05	0.12000D 03
0.22151D 02	0.86404D 05	0.27704D 05	0.13000D 03
0.22217D 02	0.87920D 05	0.29044D 05	0.14000C 03
0.21826D 02	0.89203D 05	0.31365D 05	0.15000D 03
0.21121D 02	0.90432D 05	0.34461D 05	0.16000D 03
0.20193D 02	0.91767D 05	0.38256D 05	0.17000D 03
0.19023D 02	0.93141D 05	0.42730D 05	0.18000D 03
0.17521D 02	0.94317D 05	0.47887D 05	0.19000D 03
0.15607D 02	0.95100D 05	0.53741D 05	0.20000D 03
0.13303D 02	0.95569D 05	0.60315D 05	0.21000D 03
0.10671D 02	0.95909D 05	0.67632D 05	0.22000D 03
0.77484C 01	0.96254D 05	0.75720D 05	0.23000C 03
0.45222D 01	0.96592D 05	0.84608D 05	0.24000C 03
0.96874C 00	0.96891D 05	0.94324D 05	0.25000D 03
0.99151D-10	0.96959D 05	0.96959D 05	0.25257D 03

APPENDIX D - Path Performance Program

User Instructions

The program is written in FORTRAN IV and is designed for execution in double precision on an IBM 370/165. For convenient description, the program is divided into five parts as follows:

(1) Mainline - This section handles overall program control, reads all input data, converts the variable units so that they are consistent for execution but convenient for input and output. Based on the initial input, it adjusts the value of sea level density so that for the first point lift equals weight to within maximum machine accuracy. This is necessary for subsequent calculations to yield accurate values of γ and $\dot{\gamma}$. If an adjustment in ρ_o of more than 5% is needed, the input data is considered inconsistent and execution stops. Therefore, the user should always adjust the input data so that lift and weight are initially equal. The mainline also prints out the integrated solution and tests the results for variables exceeding upper or lower limits.

(2) Subroutine SPLINE - This subroutine provides maximum power available, as a function of a h and V, for use in testing the calculated power or possibly as an input when power is specified as P_{max}. A detailed description of the spline procedure and calculation of $P_{max}(h,V)$ is given in the section entitled Computerization Procedure for Point Performance.

(3) Subroutine F - Corresponding to a particular pair of specified variables, this subroutine calculates both derivatives of the variables to be integrated and updates the algebraic parameters at each integration point.

(4) Thirteen FUNCTION Subprograms - These subprograms supply values for the specified variables and their derivatives throughout the integration region. The specified variables may be functions of one or more different flight parameters.

(5) Subroutine TRENOR - This subroutine integrates the general equations using a modified Runge-Kutta Predictor-Corrector technique which is described both in the Path Performance section and in Appendix G. This subroutine also adjusts the integration step size by halving or doubling on the basis of an error criterion.

Throughout this program, the subscripted variable Y is used to denote any of the five integrated variables. Their positions are defined as follows:

Y(1) = range, x
Y(2) = weight, W
Y(3) = altitude, h
Y(4) = flight path angle, γ
Y(5) = velocity, V .

Once a suitable set of specified variables has been chosen, the user selects the associated Key number from Table D-1.

Key	Specified Variables	FUNCTION Subprograms Utilized
1	h, V	H, DH, DDH, V, DV
2	h, γ	H, DH, DDH, GAM, DGAM
3	h, α	H, DH, DDH, ALPHA, DALPHA
4	h, W	H, DH, DDH, W, DW
5	h, P	H, DH, DDH, P
6	V, γ	V, DV, GAM, DGAM
7	V, α	V, DV, ALPHA
8	V, W	V, DV, W, DW
9	V, P	V, DV, P
10	γ, α	GAM, DGAM, DDGAM, ALPHA, DALPHA
11	γ, W	GAM, DGAM, W, DW
12	γ, P	GAM, DGAM, P
13	α, W	ALPHA, W, DW
14	α, P	ALPHA, P

Table D-1. Relation between Key numbers, specified variables, and FUNCTION subprograms.

Based on the Key numbers this program selects the variables which will be integrated and those to be specified. Corresponding to a Key number, the user must provide relations for the associated FUNCTION subprograms as shown in Table D-1. Even though each specified variable and its derivatives are implicitly functions of time (the independent variable), they may be explicit functions of other parameters as well. If this is the case, it is mandatory that the argument lists of both the FUNCTION subprograms and of the calling statement in subroutine F be made compatible. The following two examples will illustrate this procedure.

Example (1). Choose velocity and angle of attack as the specified variables. From Table D-1, this pair has a Key value of seven. For Key = 7, Table D-1 indicates utilization of subprograms V, DV, and ALPHA. Let velocity equal 151 feet per second and angle of attack equal that for maximum lift to drag ratio, as in Case (3) of Table 5. The FUNCTION subprograms now appear as:

```
      FUNCTION V(T)
      IMPLICIT REAL*8(A-H,O-Z)
      V = 151.D0
      RETURN
      END

      FUNCTION DV(T)
      IMPLICIT REAL*8(A-H,O-Z)
      DV = 0.0D0
      RETURN
      END

      FUNCTION ALPHA(T)
      IMPLICIT REAL*8(A-H,O-Z)
      ALPHA = .10258D0
      RETURN
      END
```

The FUNCTION calls of subroutine F in the Key = 7 section would then read:

```
   29 Y(5) = V(T)
      ALPHA1 = ALPHA(T)
      P1 = Y(2)*Y(5)*DV(T)/G+ • • •
```

Example (2). This example is more complex and is designed to help demonstrate the program's flexibility. Let velocity and flight path angle be the specified variables. Table D-1 indicates Key = 6 and the needed FUNCTION subprograms are V, DV, GAM, DGAM. Let V, $\dot{V}$, γ, and $\dot{\gamma}$ be those for the landing analysis described by Equations (54)-(59). The variables are as follows:

$$\gamma = \begin{cases} -.04363 \text{ radians} & 17.45 < H \leq 1500 \\ -\tan^{-1}\left(\dfrac{X - 34758.19}{H - 18333.1}\right) & 0 \leq H \leq 17.45 \end{cases}$$

$$\dot{\gamma} = \begin{cases} 0.0 & \\ & \text{for all } H \\ \approx 0.0 & \end{cases}$$

$$V = \begin{cases} 140.0 & 17.45 < H \le 1500 \\ \left(\frac{50}{17.45}\right) H + 90. & 0 \le H \le 17.45 \end{cases}$$

and since

$$\dot{V} = \frac{dV}{dt} = \frac{dV}{dh}\frac{dh}{dt} = \frac{50}{17.45}\dot{h} = \frac{50}{17.45} V \sin\gamma$$

Thus,

$$\dot{V} = \begin{cases} 0.0 & 17.45 < H \le 1500 \\ \frac{50}{17.45} V \sin\gamma & 0 \le H \le 17.45 \end{cases} .$$

The FUNCTION subprograms corresponding to each of these variables are as follows:

```
  FUNCTION GAM(X,H)
      .
      .
  IF(H.LE.17.45D0) GO TO 1
  GAM = -.04363D0
  RETURN
1 GAM = -DATAN((X-34758.19D0)/(H-18333.1D0))
  RETURN
  END

  FUNCTION DGAM(T)
      .
      .
  DGAM = 0.0D0
  RETURN
  END

  FUNCTION V(H)
      .
      .
  IF(H.LE.17.45D0) GO TO 1
  V = 140.D0
  RETURN
1 V = 50.D0*H/17.45D0+90.D0
  RETURN
  END
```

```
      FUNCTION DV(H,V,GAM)
          .
          .
      IF(H.LE.17.45DO) GO TO 1
      DV = 0.0DO
      RETURN
    1 DV = 50.DO*V*DSIN(GAM)/17.45DO
      RETURN
      END
```

For these variables the FUNCTION calls of subroutine F in Key = 6 section should read as follows:

```
   28 Y(5) = V(Y(5))
      Y(4) = GAM(Y(1),Y(3))
      CL = (DGAM(T)+G . . .
      P1 = Y(2)*Y(5)*DV(Y(3),Y(5),Y(4))/G+ . . .
```

These two examples in addition to those of the sample program, which appears later in this Appendix, serve to illustrate the steps necessary for supplying the specified variables.

For analyzing a particular aircraft, this program requires input data which may be grouped into three categories. Each input variable is defined in detail on the first page of the program listing.

Group (1). The first group, composed of all information necessary to calculate maximum power available as a function of velocity and altitude, is identical with the initial set of data cards for the point performance program. The number N of power available versus velocity data points to be specified; the control parameter ISUP which designates whether the engine is supercharged (ISUP = 1) or unsupercharged (ISUP = 0) and the reference altitude HREF (feet) which is the altitude at which the power versus velocity data points are obtained (for a supercharged aircraft HREF must be sea level in this program) are read from the first data card. The next N cards contain data points of maximum power available PA (ft-lbs per sec) versus velocity VA (ft per sec) with one data point per card, and power specified first. Thus the first N + 1 cards of the two programs have an identical purpose.

Group (2). The second group consists of variables which control the integration of the path performance equations. The value of KEY indicates those variables which will be integrated and those to be specified. PRINT gives the frequency of print out. MAXHLV limits the net number of times for halving the step size. ACCHLV and ACCDBL govern the halving and doubling of the step size and HMAX is the maximum step size permitted.

Group (3). The third group includes initial conditions for the trajectory, pertinent aircraft characteristics, the maximum time for the trajectory, and other necessary parameters. These variables are defined, complete with units, in the program listing.

Table D-2 gives the format specification for the input of all the necessary data.

N	ISUP	HREF
I10	I10	D20.13

PA(1)	VA(1)
D20.13	D20.13
.	.
.	.
.	.

PA(N)	VA(N)
D20.13	D20.13

KEY	IPRINT	MAXHLV	ACCHLV	ACCDBL	HMAX
I10	I10	I10	D15.8	D15.8	D15.8

H1	V1	GAM1	ALPHA1	W1	P1	X1	C
D10.3	D10.3	D10.3	D10.3	D10.3	D10.3	D10.3	D10.3

TDEL	TMAX	G	S	RHO	CLAO	CLA	CDO
D10.3	D10.3	D10.3	D10.3	D10.3	D10.3	D10.3	D10.3

E	AR	VMIN	WEMPTY	EX
D10.3	D10.3	D10.3	D10.3	D10.5

Table D-2. Input format specification for data of the Path Performance Program.

This program is designed so that the data of Group (1) (for computing $P_{max}(h,V)$) is read only once during the program execution. For a particular vehicle with P_{max} described by Group (1) data, sets of data from Groups (2) and (3) may be repeatedly read for calculating several different trajectories. Upon completion of a single trajectory, the program returns to the statement where KEY, IPRINT, . . . are read. If KEY is any integer between 1 and 14 the program reads a new set of Group (2) and (3) data. This process may be often repeated. After the last trajectory a card with KEY equal zero is inserted to stop the program execution.

The program listing and sample output which appear at the end of this appendix are used to illustrate the integration of two different trajectories during one program execution. These two examples will adequately illustrate the program's operation, even though several other cases could have easily been included. Compatibility of the FUNCTION subprogram argument lists with their calling statements may be observed in the program listing. The sample output resulted from the program's execution of the example data set in Table D-3. The following discussion describes collection of input data for each example with the total data set presented in Table D-3. The airframe for these examples is presented in Figure 2, and the power (PA) versus velocity (VA) points were obtained from Appendix F.

Example (1). An aircraft initially weighing 2650 pounds and having a parabolic drag polar is to fly with angle of attack for maximum lift to drag ratio at a constant altitude of 10000 feet.

Let

$$C_D = C_{D_o} + \frac{{C_L}^2}{\pi eAR} = .0269 + \frac{{C_L}^2}{\pi(.98)(7.378)}$$

thus

$$\alpha_{(L/D)_{max}} = .10258 \text{ radians} = 5.8778 \text{ degrees.}$$

Also,

$$C_L = C_{L_{(\alpha=0)}} + C_{L_\alpha}\alpha = .309 + 4.608(.10258) = .7817$$

and

$$C_D = .0538 \quad .$$

At the initial point lift must equal weight, so

$$\tfrac{1}{2}\rho(h_o)V_o^2 S\ C_L = W_o$$

or

$$V_o = \left(\frac{2W_o}{\rho(h_o)C_L S}\right)^{\frac{1}{2}} = \left(\frac{2(2650)}{(.001755)(.7817)(174)}\right)^{\frac{1}{2}}$$

$$V_o = 149.0 \text{ ft/sec}$$

and

$$P_o = \tfrac{1}{2}\rho(h_o)C_D V_o^3 = 27173.0 \text{ ft-lbs/sec} = 49.4 \text{ hp} \quad .$$

Thus the initial conditions are as follows:

$h_o = 10000.$ $\quad$ $W_o = 2650.$
$V_o = 149.$ $\quad$ $P_o = 49.4$
$\gamma_o = 0.0$ $\quad$ $x_o = 0.0$
$\alpha_o = 5.8778$

Values for the other necessary parameters are as follows:

c	= 0.6	C_{D_o}	= .0269
TDEL	= 0.5	E	= .98
TMAX	= 480.0	AR	= 7.378
G	= 32.2	VMIN	= 35.0
S	= 174.	WEMPTY	= 2150.
ρ_o	= .00238	EX	= 2.0
C_{L_α}	= 4.608	$C_{L(\alpha=0)}$	= 0.309

For α and h specified, Table D-1 indicates that KEY = 3. The correct FUNCTION subprograms associated with this value are shown in the program listing. Typical values of the other parameters are as follows:

IPRINT	= 10	ACCHLV	= .001
MAXHLV	= 2	ACCDBL	= .00002

Since these inputs produce rather smooth solutions a maximum step size of 60 seconds (HMAX = 60.) is employed.

The second example will consider an airframe slightly different from that of the first example but described by the same maximum power curves.

Example (2). A 2700 pound aircraft initially flying with velocity of 120 ft/sec at sea level and having a non-parabolic drag polar is to climb under full throttle for 30 minutes with a constant flight path angle of 1.5 degrees (.026178 radians). Let

$$C_D = .02987 + .07276\ C_L^{2.991}$$

then

$$C_{D_o} = .02987$$

and

$$\frac{1}{\pi e AR} = .07276 \qquad \text{or} \qquad e = \frac{1}{\pi(7.378)(.07276)} = .593\ .$$

Since lift and weight must be made equal at the first point, the initial angle of attack is computed as follows:

$$C_L = \frac{2W_o}{\rho(h_o)V_o^2 S} = \frac{2(2700)}{(.00238)(120)^2\ 174} = .9055$$

and

$$\alpha_o = (C_L - C_{L_{(\alpha=0)}})/C_{L_\alpha} = (.9055 - .309)/4.608$$

$$\alpha_o = .12945 \text{ radians} = 7.42 \text{ degrees}$$

$$P_o = P_{max}(h_o, V_o) = P_{max}(0.0, 120) = 153.5 \text{ hp}$$

The complete initial conditions are as follows:

$h_o = 0.0$	$W_o = 2700.$
$V_o = 120.$	$P_o = 153.5$
$\gamma_o = 1.5$	$x_o = 0.0$
$\alpha_o = 7.42$	

Other necessary parameters are:

$c = 0.6$	$C_{D_o} = .02987$
TDEL = 0.5	E = .593
TMAX = 30.0	AR = 7.378
G = 32.2	VMIN = 35.0
S = 174.	WEMPTY = 2150.
$\rho_o = .00238$	EX = 2.991
$C_{L_\alpha} = 4.608$	$C_{L_{(\alpha=0)}} = 0.309$

For γ and P as specified variables, Table D-1 indicates KEY = 12. The correct FUNCTION subprograms associated with this value are shown in the program listing. Typical values of the other control parameters are as follows:

IPRINT = 5	ACCHLV = .001
MAXHLV = 2	ACCDBL = .00002

Since flight path angle is restricted to be a constant, no oscillations are present and HMAX is set equal to 30 seconds. The input data for these two examples is presented in Table D-3. The solution time histories for these two examples are presented in the sample output.

The overall program has some limitations which should be brought to the user's attention. Several of these could be overcome by increasing the program's complexity, but for the analysis of light aircraft this was deemed unnecessary. The lift coefficient, C_L, was restricted to lie between 0 and 15 so as to prevent the inclusion of a general root solver. A maximum flight path angle of one radian was imposed solely as a check point. For most Key numbers the program may be used for flight path angles having magnitudes of nearly 90 degrees. In the derivation of the general equations of motion, power was assumed independent of angle of attack. If this restriction is

KEY							LAST CARD
0							

E	AR	VMIN	WEMPTY	EX
5.93D-01	7.378D0	3.5D+01	2.15D+03	2.991D0

TDEL	TMAX	G	S	RHO	CLA0	CLA	CD0
0.5D0	3.D+01	3.22D+01	1.74D+02	2.38D-03	3.09D-01	4.608D0	2.987D-02

H1	V1	GAM1	ALPHA1	W1	P1	X1	C
0.0D0	1.2D+02	1.5D0	7.42D0	2.7D+03	1.535D+02	0.0D0	0.6D0

KEY	IPRINT	MAXHLV	ACCHLV	ACCDBL	HMAX
12	5	2	1.D-03	2.D-05	3.D+01

E	AR	VMIN	WEMPTY	EX
0.98D0	7.378D0	3.5D+01	2.15D+03	2.D0

TDEL	TMAX	G	S	RHO	CLA0	CLA	CD0
0.5D0	4.8D+02	3.22D+01	1.74D+02	2.38D-03	3.09D-01	4.608D0	2.69D-02

H1	V1	GAM1	ALPHA1	W1	P1	X1	C
1.D+04	1.49D+02	0.0D0	5.8778D0	2.65D+03	4.94D+01	0.0D0	0.6D0

KEY	IPRINT	MAXHLV	ACCHLV	ACCDBL	HMAX
3	10	2	1.D-03	2.D-05	6.D+01

$(PA)_{15}$	$(VA)_{15}$
9.947D+04	3.8266D+02
. . .	. . .

$(PA)_2$	$(VA)_2$
2.915D+04	2.733D+01

$(PA)_1$	$(VA)_1$
0.0D0	0.0D0

N	ISUP	HREF	FIRST CARD
15	0	0.0D0	

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Table D-3. Input data for example given in Program Listing and Sample Output.

too severe, the user may modify the equations in subroutine F under a particular Key number to provide for any variation in power with angle of attack. Finally, the program structure is such that during one execution the same variable may not be specified in two different ways, since this would necessitate two different FUNCTION subprograms for the same calling statements. However, a variable may be incremented for several similar trajectories by simply passing a counter in the argument list of the FUNCTION call. For most light aircraft studies the aforementioned restrictions pose no problem to the user.

The following discussion contains "trouble shooting" aids for the user who has a new problem for the program to solve. If the program message reads:

(1) "INCONSISTENT INPUT DATA, EXECUTION CEASED FOR THAT KEY NUMBER" - Lift and weight were not equal or the program attempted to calculate a C_D less than C_{D_O} at the initial point.

(2) "VALUE OF MAXHLV WAS EXCEEDED, EXECUTION STOPPED" - The value of MAXHLV should be increased and/or use a smaller initial step size (TDEL).

(3) "ABSOLUTE VALUE OF FLIGHT PATH ANGLE BECAME GREATER THAN 1.0 RADIAN" - If this error suddenly appears when the actual printed time history is small in magnitude but oscillatory, the corrective action if to decrease HMAX so as to prevent the rapid halving and doubling of the step size.

(4) "ALTITUDE BECAME NEGATIVE" - This may occur when the actual trajectory closely approaches sea level. If the initial altitude is zero, then the start of integration may cause a slight oscillation which gives an erroneous negative altitude. In this case, increase h_O to perhaps 50 feet.

For maximum integration efficiency, values for ACCHLV and ACCDBL may be adjusted based on the experience of previous executions to produce a minimum of halving and doubling the step size.

The following listing and sample output complete this description of the Path Performance Program.

Listing

```
C
C
C DEFINITION OF INPUT VARIABLES:
C
C     N - NUMBER OF POINTS ON PA VS. VA CURVE.
C     ISUP - CONTROL VARIABLE FOR SUPERCHARGED ENGINE
C     ISUP=0 - ENGINE NOT SUPERCHARGED
C     ISUP=1 - ENGINE SUPERCHARGED AND REFERENCE ALTITUDE IS SEA LEVEL
C     HREF - REFERENCE ALTITUDE AT WHICH POWER AVAILABLE VS. VELOCITY
C       CURVE IS GIVEN
C     PA - POWER AVAILABLE IN FOOT - POUNDS PER SECOND
C     VA - VELOCITY IN FEET PER SECOND.
C     KEY - CODE NUMBER DENOTING CASE TO BE COMPUTED.
C     KEY=0 - TERMINATES THE PROGRAM
C     KEY=1 - ALTITUDE & VELOCITY ARE SPECIFIED
C     KEY=2 - ALTITUDE & FLIGHT PATH ANGLE ARE SPECIFIED
C     KEY=3 - ALTITUDE & ANGLE OF ATTACK ARE SPECIFIED
C     KEY=4 - ALTITUDE & WEIGHT ARE SPECIFIED
C     KEY=5 - ALTITUDE & POWER ARE SPECIFIED
C     KEY=6 - VELOCITY & FLIGHT PATH ANGLE ARE SPECIFIED
C     KEY=7 - VELOCITY & ANGLE OF ATTACK ARE SPECIFIED
C     KEY=8 - VELOCITY & WEIGHT ARE SPECIFIED
C     KEY=9 - VELOCITY & POWER ARE SPECIFIED
C     KEY=10 - FLIGHT PATH ANGLE & ANGLE OF ATTACK ARE SPECIFIED
C     KEY=11 - FLIGHT PATH ANGLE & WEIGHT ARE SPECIFIED
C     KEY=12 - FLIGHT PATH ANGLE & POWER ARE SPECIFIED
C     KEY=13 - ANGLE OF ATTACK & WEIGHT ARE SPECIFIED
C     KEY=14 - ANGLE OF ATTACK & POWER ARE SPECIFIED
C
C     IPRINT - NUMBER OF INTEGRATIONS BETWEEN VARIABLE PRINT OUT
C       (I.E. IPRINT=1 IMPLIES RESULTS PRINTED FOR EACH INTEGRATION STEP)
C
C     MAXHLV - NET NUMBER OF TIMES THE INTEGRATION STEP MAY HALVE BEFORE
C       EXECUTION IS STOPPED FOR THAT KEY NUMBER.
C
C     ACCHLV - INTEGRATION STEPSIZE IS HALVED WHEN ERROR ESTIMATE
C       EXCEEDS THIS VALUE
C
C     ACCDBL - INTEGRATION STEP IS DOUBLED WHEN ERROR ESTIMATE
C       FALLS BELOW THIS VALUE.
C
C     HMAX - MAXIMUM INTEGRATION STEPSIZE IN SECONDS.
C
C     H1 - INITIAL VALUE OF ALTITUDE IN FEET.
C     V1 - INITIAL VALUE OF VELOCITY IN FEET/SEC.
C     GAM1 - INITIAL VALUE OF FLIGHT PATH ANGLE IN DEGREES.
C     ALPHA1 - INITIAL VALUE OF ANGLE OF ATTACK IN DEGREES.
C     W1 - INITIAL VALUE OF WEIGHT IN POUNDS.
C     P1 - INITIAL VALUE OF POWER IN HORSEPOWER.
C     X1 - INITIAL VALUE OF DISTANCE TRAVELED ALONG THE GROUND IN MILES.
C     C - PROPORTIONALITY CONSTANT BETWEEN POWER AND CHANGE IN WEIGHT;
C       C HAS UNITS OF POUNDS PER HORSEPOWER - HOUR.
C     TDEL - INITIAL VALUE OF INTEGRATION STEPSIZE IN SECONDS.
C     TMAX - MAXIMUM VALUE OF TIME TO BE CONSIDERED IN MINUTES.
C     G - 32.2 FT/SEC*SEC.
C     S - AIRCRAFT WING AREA IN SQUARE FEET.
C     RHO - DENSITY OF AIR AT SEA LEVEL, 0.00238 SLUGS/FT**3.
C     CLAO - LIFT COEFFICIENT AT ZERO ANGLE OF ATTACK.
C     CLA - LIFT CURVE SLOPE PER RADIAN.
C     CDO - PARASITE DRAG COEFFICIENT
C     E - OSWALD'S EFFICIENCY FACTOR.
C     AR - WING ASPECT RATIO.
C     VMIN - AIRCRAFT MINIMUM SPEED IN FEET PER SECOND.
C     WEMPTY - AIRCRAFT WEIGHT WITH ZERO FUEL IN POUNDS.
C     EX - EXPONENTIAL POWER TO WHICH LIFT COEFFICIENT IS RAISED IN
C       ESTIMATING INDUCED DRAG.
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Y(5),DY(5),IKEY(14,6),PA(20),VA(20)
      EXTERNAL H,DH,DDH,V,DV,GAM,DGAM,ALPHA,DALPHA,W,DW,P,F
      COMMON KEY,JKEY(5)
      COMMON /PARAM/ALPHA1,P1,CD,CL,C,S,RHO,CLAO,CLA,CDO,PI,E,AR,G,EX
      DATA IKEY/14*1,3*2,5,3*2,3,2,2,3,2,3,2,0,5,5,0,5,3,3,4,3,3,5,3,4,3
     1,0,0,0,0,0,0,4,0,4,5,0,5,5,4,13*0,5,2,3,3,2,3,3,4,3,4,4,3,4,4,5/
    1 FORMAT (10(2X,G11.4))
      READ (1,2) N,ISUP,HREF
    2 FORMAT (2I10,D20.13)
      READ (1,3) (PA(I),VA(I),I=1,N)
    3 FORMAT (2D20.13)
      IF (ISUP.NE.0) GO TO 5
      WRITE (3,4) HREF
    4 FORMAT (1H1//1X,52X,'POWER AVAILABLE VS. VELOCITY'/1X,47X,'REFEREN
     1CE ALTITUDE =',D12.5,1X,'FEET'//1X,50X,'PA(FT-LBS/SEC)',6X,'VA(FT/
     1SEC)')
      GO TO 7
    5 WRITE (3,6) HREF
    6 FORMAT (1H1//1X,52X,'POWER AVAILABLE VS. VELOCITY'/1X,56X,'SUPERCH
     1ARGED ENGINE'/1X,47X,'REFERENCE ALTITUDE =',D12.5,' FEET'//1X,50X,
     1'PA(FT-LBS/SEC)',7X,'V(FT/SEC)')
    7 WRITE (3,8) (PA(I),VA(I),I=1,N)
    8 FORMAT (1X,51X,D12.5,6X,D12.5)
      CALL SPLINE(N,PA,VA,ISUP,HREF)
    9 READ (1,10) KEY,IPRINT,MAXHLV,ACCHLV,ACCDBL,HMAX
   10 FORMAT (3I10,3(D15.8))
      IF (KEY.EQ.0.OR.KEY.GE.15) CALL EXIT
      READ (1,11) H1,V1,GAM1,ALPHA1,W1,P1,X1,C,TDEL,TMAX,G,S,RHO,CLAO,CL
     1A,CDO,E,AR,VMIN,WEMPTY,EX
   11 FORMAT (8D10.3)
      WRITE (3,12) H1,V1,GAM1,ALPHA1,W1,P1,X1,C,TDEL,TMAX,G,S,RHO,CLAO,C
     1LA,CDO,E,AR,KEY,IPRINT,MAXHLV,ACCHLV,ACCDBL,HMAX,VMIN,WEMPTY,EX
   12 FORMAT ('1',1X,'H1 = ',G11.4,9X,'V1 = ',G11.4,9X,'GAM1 = ',G11.4,7
     1X,'ALPHA1 = ',G11.4,5X,'W1 = ',G11.4,//,2X,'P1 = ',G11.4,9X,'X1 =
     1',G11.4,9X,'C = ',G11.4,10X,'TDEL = ',G11.4,7X,'TMAX = ',G11.4,//,
     12X,'G = ',G11.4,10X,'S = ',G11.4,10X,'RHO = ',G11.4,8X,'CLAO = ',G
     111.4,7X,'CLA = ',G11.4,//,2X,'CDO = ',G11.4,8X,'E = ',G11.4,10X,'A
     1R = ',G11.4,9X,'KEY = ',I2,17X,'IPRINT = ',I3,//,2X,'MAXHLV = ',I3
     1,13X,'ACCHLV = ',G11.4,5X,'ACCDBL = ',G11.4,5X,'HMAX = ',G11.4,7X,
     1'VMIN = ',G11.4,//,2X,'WEMPTY = ',G11.4,5X,'EX = ',G11.4,///)
      WRITE (3,13)
   13 FORMAT (4X,'TIME',7X,'ALTITUDE',5X,'VELOCITY',5X,'GAMMA',10X,'ALPH
     1A',8X,'CL',13X,'CD',7X,'WEIGHT',8X,'POWER',9X,'RANGE',/,5X,'(MIN)'
     1,8X,'(FT)',7X,'(FT/SEC)',5X,'(DEG)',10X,'(DEG)',31X,'(POUNDS)',8X,
     1'(HP)',8X,'(MILES)',/)
C
C     CONVERT VARIABLES SO THAT UNITS ARE CONSISTENT FOR COMPUTATION.
C
      TMAX=60.D0*TMAX
      GAM1=GAM1/57.29578
      ALPHA1=ALPHA1/57.29578
      X1=5280.D0*X1
      P1=P1*550.D0
      C=C/(550.D0*3600.D0)
      PI=3.1415926535D0
C
C     SPECIFY THE MAXIMUM GAMMA AS ONE RADIAN.
```

```
C     INITIALIZE THE INTEGRATION INTERVAL TO ZERO TIME.
C     INITIALIZE COUNTERS:  K FOR DETERMINING PRINTOUT FREQUENCY, ISET
C       FOR SWITCHING THE MODIFIED RUNGE-KUTTA ON OR OFF, AND KNTHLV FOR
C       CETERMINING WHETHER MAXHLV HAS BEEN EXCEEDED.
C
      GAMMAX=1.0D0
      T=0.0
      K=0
      ISET=0
      KNTHLV=0
C
C     THE FOLLOWING SIX CAROS ADJUST THE VALUE OF SEA LEVEL DENSITY SO
C       THAT THE AIRCRAFT'S LIFT AND WEIGHT ARE EQUAL AT THE INITIAL
C       POINT.  THIS PREVENTS UNNECESSARY ERROR IN SUBSEQUENT EVALUATION
C       OF D(GAMMA)/D(T).  IF THIS CHANGE IN RHO IS IN EXCESS OF FIVE
C       PERCENT, THEN THE INPUT DATA IS DEEMED INCONSISTENT.
C
      FH=(1.0-6.86*1.D-6*H1)**4.26
      CL=CLA0+CLA*ALPHA1
      RHONEW=2.0D0*W1/(S*V1*V1*CL*FH)
      TEST=(RHONEW-RHO)/RHO
      IF (DABS(TEST).GT.0.05) GO TO 16
      RHO=RHONEW
      CD=CD0+CL**EX/(PI*E*AR)
      Y(1)=X1
      Y(2)=W1
      Y(3)=H1
      Y(4)=GAM1
      Y(5)=V1
      NEON=IKEY(KEY,6)
      DO 14 J=1,NEON
   14 JKEY(J)=IKEY(KEY,J)
      CALL F(T,Y,DY,&16)
      TIME=T/60.D0
      GAMDEG=Y(4)*57.29578
      ALPDEG=ALPHA1*57.29578
      POWER=P1/550.D0
      RANGE=Y(1)/5280.D0
      WRITE (3,1) TIME,Y(3),Y(5),GAMDEG,ALPDEG,CL,CD,Y(2),POWER,RANGE
   15 CALL TRENOR(NEON,TDEL,T,Y,DY,F,HMAX,ACCHLV,ACCDBL,MAXHLV,KNTHLV,IS
     1ET)
C
C     TEST VARIABLES TO PREVENT THEM FROM EXCEEDING SPECIFIED LIMITS.
C
      IF (Y(2).LT.WEMPTY) GO TO 18
      IF (Y(3).LT.0.0D0) GO TO 20
      IF (DABS(Y(4)).GT.GAMMAX) GO TO 22
      IF (Y(5).LT.VMIN) GO TO 24
      CALL POMAX(Y(5),Y(3),PMAX)
      IF (P1.GT.PMAX) GO TO 26
      K=K+1
C
C     IF MAXHLV IS RETURNED AS -1, THEN THE NET NUMBER OF TIMES THE
C       INTEGRATION STEP-SIZE WAS HALVED HAS EXCEEDED MAXHLV AND
C.      EXECUTION IS CEASED FOR THAT KEY NUMBER.
C
      IF (MAXHLV.EQ.-1) GO TO 9
      IF (K.LT.IPRINT.AND.T.LE.TMAX) GO TO 15
      TIME=T/60.D0
      GAMDEG=Y(4)*57.29578
      ALPDEG=ALPHA1*57.29578
      POWER=P1/550.D0
      RANGE=Y(1)/5280.D0
      WRITE (3,1) TIME,Y(3),Y(5),GAMDEG,ALPDEG,CL,CD,Y(2),POWER,RANGE
      K=0
      IF (T.LE.TMAX) GO TO 15
      GO TO 9
   16 WRITE (3,17)
   17 FORMAT (1X,'INCONSISTENT INPUT DATA, EXECUTION CEASED FOR THAT KEY
     1 NUMBER')
      GO TO 9
   18 WRITE (3,19)
   19 FORMAT (1X,'AIRCRAFT WEIGHT BECAME LESS THAN WEIGHT FOR ZERO FUEL'
     1)
      GO TO 9
   20 WRITE (3,21)
   21 FORMAT (1X,'ALTITUDE BECAME NEGATIVE')
      GO TO 9
   22 WRITE (3,23)
   23 FORMAT (1X,'ABSOLUTE VALUE OF FLIGHT PATH ANGLE BECAME GREATER THA
     1N 1.0 RADIAN')
      GO TO 9
   24 WRITE (3,25)
   25 FORMAT (1X,'VELOCITY BECAME LESS THAN THE MINIMUM SPEED')
      GO TO 9
   26 WRITE (3,27) PMAX
   27 FORMAT (1X,'POWER BECAME GREATER THAN A PMAX OF ',D12.5,' IN FT-L
     1BS/SEC')
      GO TO 9
      END
```

```
      SUBROUTINE SPLINE(N,Y,X,ISUP,HREF)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION AA(4,20),H(20),Q(20),U(20),X(20),Y(20)
C
C     THIS SUBROUTINE CALCULATES THE MAXIMUM POWER AVAILABLE AS A
C       FUNCTION OF ALTITUDE AND VELOCITY.
C
      NM1=N-1
      DO 1 I=1,NM1
    1 H(I)=X(I+1)-X(I)
      Q(1)=-31.0D0/32.0D0
      H1=H(1)
      H2=H(2)
      H3=H(3)
      U(1)=Y(1)*(32.0D0*H1+42.0D0*H2+21.0D0*H3)/(H1+H2)/(H1+H2+H3)-Y(2)*
     1(11.0D0*H1+42.0D0*H2+21.0D0*H3)/(H2+H3)/H2+Y(3)*H1*(11.0D0*H1+21.0
     1D0*(H2+H3))/(H1+H2)/H2/H3-Y(4)*H1*(11.0D0*H1+21.0D0*H2)/(H2+H3)/(H
     11+H2+H3)/H3
      U(1)=3.0D0*U(1)/H1/16.D0
      HM=H(1)
      YY=Y(2)
      YM=Y(1)
      DO 2 I=2,NM1
      HH=H(I)
      YP=Y(I+1)
      D=3.0D0*((YP-YY)/HH-(YY-YM)/HM)/(HH+HM)
      C=0.5D0*HH/(HH+HM)
      A=0.5D0-C
      P=A*Q(I-1)+1.0D0
      Q(I)=-C/P
      U(I)=(D-A*U(I-1))/P
      HM=HH
      YM=YY
    2 YY=YP
      A=31.0D0/32.0D0
```

```
      P=A*Q(N-1)+1.0D0
      H1=H(NM1)
      H2=H(NM1-1)
      H3=H(NM1-2)
      C=Y(N)*(32.0D0*H1+42.0D0*H2+21.0D0*H3)/(H1+H2)/(H1+H2+H3)-Y(NM1)*(
     111.0D0*H1+42.0D0*H2+21.0D0*H3)/(H2+H3)/H2+Y(NM1-1)*H1*(11.0D0*H1+2
     11.0D0*(H2+H3))/(H1+H2)/H2/H3-Y(NM1-2)*H1*(11.0D0*H1+21.0D0*H2)/(H2
     1+H3)/(H1+H2+H3)/H3
      D=3.0D0*D/H1/16.0D0
      U(N)=(D-A*U(N-1))/P
      DO 3 J=1,NM1
      I=N-J
    3 U(I)=Q(I)*U(I+1)+U(I)
      UU=U(1)
      XX=X(1)
      YY=Y(1)
      DO 4 I=1,NM1
      UP=U(I+1)
      XP=X(I+1)
      YP=Y(I+1)
      HH=H(I)
      AA(1,I)=(UP-UU)/HH/6.0D0
      AA(2,I)=0.5D0*(XP*UU-XX*UP)/HH
      AA(3,I)=0.5D0*(UP*XX*XX-UU*XP*XP)/HH+(UU-UP)*HH/6.0D0+(YP-YY)/HH
      AA(4,I)=(UU*XP*XP*XP-UP*XX*XX*XX)/HH/6.0D0+(UP*XX-UU*XP)*HH/6.0D0+
     1(YY*XP-YP*XX)/HH
      XX=XP
      UU=UP
    4 YY=YP
      SGMREF=(1.0D0-6.86D-6*HREF)**4.26D0
      RETURN
C
      ENTRY POMAX(VV,HA,PMAX)
      IF (VV.GT.X(1)) GO TO 5
      I=1
      GO TO 7
    5 DO 6 I=1,NM1
      IF (X(I).LE.VV.AND.X(I+1).GT.VV) GO TO 7
    6 CONTINUE
      I=NM1
    7 A1=AA(1,I)
      A2=AA(2,I)
      A3=AA(3,I)
      A4=AA(4,I)
      PMAX=((A1*VV+A2)*VV+A3)*VV+A4
      IF (ISUP.NE.0) RETURN
      SIGMA=(1.0D0-6.86D-6*HA)**4.26D0
      PCR=(SIGMA-0.165D0)/(SGMREF-0.165D0)
      PMAX=PMAX*PCR
      RETURN
      END


      SUBROUTINE F(T,Y,DY,*)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION Y(5),DY(5)
      EXTERNAL H,DH,DDH,V,DV,GAM,DGAM,ALPHA,DALPHA,W,DW,P
      COMMON KEY,JKEY(5)
      COMMON /PARAM/ALPHA1,P1,CD,CL,C,S,RHO,CLAO,CLA,CDO,PI,E,AR,G,EX
C
C     TRANSFER TO THE APPROPRIATE SUBROUTINE SECTION CORRESPONDING TO
C       A SPECIFIC KEY VALUE.
C
      GO TO (1,2,5,8,18,28,29,30,31,32,33,34,35,36),KEW
C
C     KEW = 1
C
    1 Y(3)=H(T)
      FH=(1.0-6.86*1.D-6*Y(3))**4.26
      Y(5)=V(T)
      Y(4)=DARSIN(DH(T)/Y(5))
      DY(4)=(DDH(T)-DV(T)*DTAN(Y(4)))/(Y(5)*DCOS(Y(4)))
      CL=(DY(4)+G*DCOS(Y(4))/Y(5))*(2.0*Y(2)/(G*RHO*S*Y(5)*FH))
      ALPHA1=(CL-CLAO)/CLA
      CD=CDO+CL**EX/(PI*E*AR)
      P1=Y(2)*Y(5)*DV(T)/G+Y(2)*Y(5)*DSIN(Y(4))+S*RHO*CD*FH*Y(5)**3/2.0
      IF (P1.LT.0.0D0)P1=0.0D0
      DY(2)=-C*P1
      GO TO 37
C
C     KEY = 2
C
    2 Y(3)=H(T)
      FH=(1.0-6.86*1.D-6*Y(3))**4.26
      Y(4)=GAM(T)
      CL=(DGAM(T)+G*DCOS(Y(4))/Y(5))*(2.0*Y(2)/(G*RHO*S*Y(5)*FH))
      ALPHA1=(CL-CLAO)/CLA
      CD=CDO+CL**EX/(PI*E*AR)
      IF (DABS(Y(4)).LT.0.000001) GO TO 3
      DY(5)=(DDH(T)-Y(5)*DGAM(T)*DCOS(Y(4)))/DSIN(Y(4))
      P1=Y(2)*Y(5)*DY(5)/G+Y(2)*Y(5)*DSIN(Y(4))+S*RHO*CD*FH*Y(5)**3/2.0
      GO TO 4
    3 IF (DABS(DGAM(T)).GT.0.000001) GO TO 4
      P1=(G*CD*Y(5)*FH-CL*Y(5)*Y(5)*2.13*(1.0-6.86*1.D-6*Y(3))**3.26*(-6
     1.86*1.D-6)*DH(T))*RHO*S*Y(5)*Y(5)/(Y(5)*Y(5)*C+2.0D0*G)
      DY(5)=G*P1/(Y(2)*Y(5))-G*S*RHO*CD*Y(5)*Y(5)*FH/(2.0*Y(2))
    4 DY(2)=-C*P1
      GO TO 37
C
C     KEW = 3
C
    5 Y(3)=H(T)
      FH=(1.0-6.86*1.D-6*Y(3))**4.26
      ALPHA1=ALPHA(T)
      CL=CLAO+CLA*ALPHA1
      CD=CDO+CL**EX/(PI*E*AR)
      Y(4)=DARSIN(DH(T)/Y(5))
      DY(4)=G*S*RHO*CL*Y(5)*FH/(2.0*Y(2))-G*DCOS(Y(4))/Y(5)
      IF (DABS(Y(4)).LT.0.00001) GO TO 6
      DY(5)=(DDH(T)-Y(5)*DY(4)*DCOS(Y(4)))/DSIN(Y(4))
      P1=(DY(5)+G*S*RHO*CD*Y(5)*Y(5)*FH/(2.0*Y(2))+G*DSIN(Y(4)))*Y(2)*Y(
     15)/G
      IF (P1.LT.0.0D0)P1=0.0D0
      GO TO 7
    6 IF (DABS(DY(4)).GT.0.1D-04) GO TO 7
      P1=(G*CD*Y(5)*FH-CL*Y(5)*Y(5)*2.13*(1.0-6.86*1.D-6*Y(3))**3.26*(-6
     1.86*1.D-6)*DH(T)-Y(5)*Y(5)*FH*CLA*DALPHA(T)/2.0D0)*RHO*S*Y(5)*Y(5)
     1/(Y(5)*Y(5)*C+2.0D0*G)
      DY(5)=G*P1/(Y(2)*Y(5))-G*S*RHO*CD*Y(5)*Y(5)*FH/(2.0*Y(2))
    7 DY(2)=-C*P1
      GO TO 37
C
C     KEW = 4
C
    8 Y(3)=H(T)
      FH=(1.0-6.86*1.D-6*Y(3))**4.26
      Y(2)=W(T)
```

```
      P1=-DW(T)/C
      Y(4)=DARSIN(DH(T)/Y(5))
      IF (DABS(Y(4)).LT.0.000001) GO TO 16
      C1=G*P1/(Y(2)*Y(5))-G*DSIN(Y(4))-DDH(T)/DSIN(Y(4))-G*DCOS(Y(4))**2
     1.0/DSIN(Y(4))
      C2=G*S*RHO*Y(5)*Y(5)*FH/(2.0*Y(2))
      B=DCOS(Y(4))/DSIN(Y(4))
      A=1.0/(PI*E*AR)
      CL1=0.0D0
      F1=A*CL1**EX-B*CL1+CD0-C1/C2
    9 CL2=CL1+.1D0
      F2=A*CL2**EX-B*CL2+CD0-C1/C2
      IF ((F1*F2).LE.0.0) GO TO 10
      F1=F2
      CL1=CL2
      IF (CL1.GT.15.D0) CALL EXIT
      GO TO 9
   10 DO 14 I=1,100
      CLROOT=CL1-F1*(CL2-CL1)/(F2-F1)
      FUN=A*CLROOT**EX-B*CLROOT+CD0-C1/C2
      IF (F1*FUN)  11,15,12
   11 CL2=CLROOT
      F2=FUN
      GO TO 13
   12 CL1=CLROOT
      F1=FUN
   13 IF (DABS(1.D0-CLROOT/CL1).LT.1.0D-03) GO TO 15
   14 CONTINUE
   15 CL=CLROOT
      GO TO 17
   16 CL=(DDH(T)+G)*2.0*Y(2)/(G*S*RHO*Y(5)*Y(5)*FH)
   17 ALPHA1=(CL-CLA0)/CLA
      CD=CD0+CL**EX/(PI*E*AR)
      DY(5)=G*P1/(Y(2)*Y(5))-G*S*RHO*CD*Y(5)*Y(5)*FH/(2.0*Y(2))-G*DSIN(Y
     1(4))
      GO TO 37
C
C     KEY = 5
C
   18 Y(3)=H(T)
      FH=(1.0-6.86*1.D-6*Y(3))**4.26
      P1=P(T)
      DY(2)=-C*P1
      Y(4)=DARSIN(DH(T)/Y(5))
      IF (DABS(Y(4)).LT.0.000001) GO TO 26
      C1=G*P1/(Y(2)*Y(5))-G*DSIN(Y(4))-DDH(T)/DSIN(Y(4))-G*DCOS(Y(4))**2
     1.0/DSIN(Y(4))
      C2=G*S*RHO*Y(5)*Y(5)*FH/(2.0*Y(2))
      B=DCOS(Y(4))/DSIN(Y(4))
      A=1.0/(PI*E*AR)
      CL1=0.0D0
      F1=A*CL1**EX-B*CL1+CD0-C1/C2
   19 CL2=CL1+.1D0
      F2=A*CL2**EX-B*CL2+CD0-C1/C2
      IF ((F1*F2).LE.0.0) GO TO 20
      F1=F2
      CL1=CL2
      IF (CL1.GT.15.D0) CALL EXIT
      GO TO 19
   20 DO 24 I=1,100
      CLROOT=CL1-F1*(CL2-CL1)/(F2-F1)
      FUN=A*CLROOT**EX-B*CLROOT+CD0-C1/C2
      IF (F1*FUN)  21,25,22
   21 CL2=CLROOT
      F2=FUN
      GO TO 23
   22 CL1=CLROOT
      F1=FUN
   23 IF (DABS(1.D0-CLROOT/CL1).LT.1.0D-C3) GO TO 25
   24 CONTINUE
   25 CL=CLROOT
      GO TO 27
   26 CL=(DDH(T)+G)*2.0*Y(2)/(G*S*RHO*Y(5)*Y(5)*FH)
   27 ALPHA1=(CL-CLA0)/CLA
      CD=CD0+CL**EX/(PI*E*AR)
      DY(5)=G*P1/(Y(2)*Y(5))-G*S*RHO*CD*Y(5)*Y(5)*FH/(2.0*Y(2))-G*DSIN(Y
     1(4))
      GO TO 37
C
C     KEY = 6
C
   28 Y(5)=V(T)
      Y(4)=GAM(T)
      FH=(1.0-6.86*1.D-6*Y(3))**4.26
      DY(3)=Y(5)*DSIN(Y(4))
      CL=(DGAM(T)+G*DCOS(Y(4))/Y(5))*(2.C*Y(2)/(G*RHO*S*Y(5)*FH))
      ALPHA1=(CL-CLA0)/CLA
      CD=CD0+CL**EX/(PI*E*AR)
      P1=Y(2)*Y(5)*DV(T)/G+Y(2)*Y(5)*DSIN(Y(4))+S*RHO*CD*FH*Y(5)**3/2.0
      IF (P1.LT.0.0D0)P1=0.0DC
      DY(2)=-C*P1
      GO TO 37
C
C     KEY = 7
C
   29 Y(5)=V(T)
      ALPHA1=ALPHA(T)
      FH=(1.0-6.86*1.D-6*Y(3))**4.26
      DY(3)=Y(5)*DSIN(Y(4))
      CL=CLA0+CLA*ALPHA1
      CD=CD0+CL**EX/(PI*E*AR)
      P1=Y(2)*Y(5)*DV(T)/G+Y(2)*Y(5)*DSIN(Y(4))+S*RHO*CD*FH*Y(5)**3/2.0
      IF (P1.LT.0.0D0)P1=0.0DC
      DY(2)=-C*P1
      DY(4)=G*S*RHO*CL*Y(5)*FH/(2.0*Y(2))-G*DCOS(Y(4))/Y(5)
      GO TO 37
C
C     KEY = 8
C
   30 Y(5)=V(T)
      Y(2)=W(T)
      FH=(1.0-6.86*1.D-6*Y(3))**4.26
      DY(3)=Y(5)*DSIN(Y(4))
      P1=-DW(T)/C
      CD=(-DV(T)+G*P1/(Y(2)*Y(5))-G*DSIN(Y(4)))*2.*Y(2)/(G*S*RHC*Y(5)*Y(
     15)*FH)
      IF ((CD-CD0).LT.0.0) RETURN 1
      CL=((CD-CD0)*PI*E*AR)**(1.0D0/EX)
      ALPHA1=(CL-CLA0)/CLA
      DY(4)=G*S*RHO*CL*Y(5)*FH/(2.0*Y(2))-G*DCOS(Y(4))/Y(5)
      GO TO 37
C
C     KEY = 9
C
   31 FH=(1.0-6.86*1.D-6*Y(3))**4.26
      Y(5)=V(T)
      P1=P(T)
      DY(3)=Y(5)*DSIN(Y(4))
```

```
      CY(2)=-C*P1
      CD=(-DV(T)+G*P1/(Y(2)*Y(5))-G*DSIN(Y(4)))*2.DO*W(2)/(G*S*RHO*Y(5)*
     1Y(5)*FH)
      IF ((CD-CDO).LT.0.0) RETURN 1
      CL=((CD-CDO)*PI*E*AR)**(1.CDO/EX)
      ALPHA1=(CL-CLAO)/CLA
      DY(4)=G*S*RHO*CL*Y(5)*FH/(2.0*Y(2))-G*DCOS(Y(4))/Y(5)
      GO TO 37
C
C     KEY = 10
C
   32 Y(4)=GAM(T)
      ALPHA1=ALPHA(T)
      FH=(1.0-6.86*1.D-6*Y(3))**4.26
      CL=CLAO+CLA*ALPHA1
      CD=CDO+CL**EX/(PI*E*AR)
      CY(3)=Y(5)*DSIN(Y(4))
      P1=(DDGAM(T)-G*S*RHO*CLA*DALPHA(T)*Y(5)*FH/(2.0*Y(2))+(G*S*RHO*CL*
     1FH/(2.*Y(2))+G*DCOS(Y(4))/(Y(5)*Y(5)))*(G*S*RHO*CD*Y(5)*Y(5)*FH/(2
     1.0*Y(2))+G*DSIN(Y(4)))-G*S*RHO*CL*Y(5)*4.26*(1.0-6.86*1.D-6*Y(3))*
     1*3.26*(-6.86*1.D-6*Y(5)*DSIN(Y(4)))/(2.C*Y(2))-G*DSIN(Y(4))*DGAM(T
     1)/Y(5))/(G*(G*S*RHO*CL*FH/(2.*Y(2))+G*DCOS(Y(4))/(Y(5)*Y(5)))/(Y(2
     1)*Y(5))+G*S*RHO*C*CL*Y(5)*FH/(2.0*Y(2)*Y(2)))
      CY(2)=-C*P1
      CY(5)=G*P1/(Y(2)*Y(5))-G*S*RHO*CD*Y(5)*Y(5)*FH/(2.0*Y(2))-G*DSIN(Y
     1(4))
      GO TO 37
C
C     KEY = 11
C
   33 Y(4)=GAM(T)
      FH=(1.0-6.86*1.D-6*Y(3))**4.26
      Y(2)=W(T)
      P1=-DW(T)/C
      DY(3)=Y(5)*DSIN(Y(4))
      CL=(DGAM(T)+G*DCOS(Y(4))/Y(5))*(2.0*Y(2)/(G*RHO*S*Y(5)*FH))
      ALPHA1=(CL-CLAO)/CLA
      CD=CDO+CL**EX/(PI*E*AR)
      DY(5)=G*P1/(Y(2)*Y(5))-G*S*RHO*CD*Y(5)*Y(5)*FH/(2.0*Y(2))-G*DSIN(Y
     1(4))
      GO TO 37
C
C     KEY = 12
C
   34 Y(4)=GAM(T)
      P1=P(Y(5),Y(3))
      FH=(1.0-6.86*1.D-6*Y(3))**4.26
      CY(2)=-C*P1
      CY(3)=Y(5)*DSIN(Y(4))
      CL=(DGAM(T)+G*DCOS(Y(4))/Y(5))*(2.0*Y(2)/(G*RHO*S*Y(5)*FH))
      ALPHA1=(CL-CLAO)/CLA
      CD=CDO+CL**EX/(PI*E*AR)
      CY(5)=G*P1/(Y(2)*Y(5))-G*S*RHO*CD*Y(5)*Y(5)*FH/(2.0*Y(2))-G*DSIN(Y
     1(4))
      GO TO 37
C
C     KEY = 13
C
   35 ALPHA1=ALPHA(T)
      FH=(1.0-6.86*1.D-6*Y(3))**4.26
      Y(2)=W(T)
      P1=-CW(T)/C
      CY(3)=Y(5)*DSIN(Y(4))
      CL=CLAO+CLA*ALPHA1
      CD=CDO+CL**EX/(PI*E*AR)
      DY(4)=G*S*RHO*CL*Y(5)*FH/(2.0*Y(2))-G*DCOS(Y(4))/Y(5)
      DY(5)=G*P1/(Y(2)*Y(5))-G*S*RHO*CD*Y(5)*Y(5)*FH/(2.0*Y(2))-G*DSIN(Y
     1(4))
      GO TO 37
C
C     KEY = 14
C
   36 ALPHA1=ALPHA(T)
      FH=(1.0-6.86*1.C-6*Y(3))**0.26
      P1=P(T)
      DY(2)=-P1*C
      DY(3)=Y(5)*DSIN(Y(4))
      CL=CLAO+CLA*ALPHA1
      CD=CDO+CL**EX/(PI*E*AR)
      DY(4)=G*S*RHO*CL*Y(5)*FH/(2.0*Y(2))-G*DCOS(Y(4))/Y(5)
      DY(5)=G*P1/(Y(2)*Y(5))-G*S*RHO*CD*Y(5)*Y(5)*FH/(2.0*Y(2))-G*DSIN(Y
     1(4))
C
   37 DY(1)=Y(5)*DCOS(Y(4))
      RETURN
      END


C     THE FOLLOWING 13 FUNCTION SUBPROGRAMS ARE USED TO SUPPLY
C       INFORMATION CONCERNING THE VARIABLES SPECIFIED FOR A PARTICULAR
C       KEY VALUE.  FOR EXAMPLE, IF KEY = 1, THEN THE FUNCTIONS H,DH,
C       DDH,V,AND DV MUST BE ACCURATELY SUPPLIED.


      FUNCTION H(T)
      IMPLICIT REAL*8(A-H,O-Z)
      H=10000.CO
      RETURN
      END


      FUNCTION DH(T)
      IMPLICIT REAL*8(A-H,O-Z)
      DH=0.0D0
      RETURN
      END


      FUNCTION DDH(T)
      IMPLICIT REAL*8(A-H,O-Z)
      DDH=0.0D0
      RETURN
      END


      FUNCTION V(T)
      IMPLICIT REAL*8(A-H,O-Z)
      V=219.D0
      RETURN
      END
```

```
      FUNCTION DV(T)
      IMPLICIT REAL*8(A-H,O-Z)
      DV=0.0D0
      RETURN
      END


      FUNCTION GAM(T)
      IMPLICIT REAL*8(A-H,O-Z)
C
C     GAM SHOULD BE RETURNED IN RADIANS.
C
      GAM=.0261780D0
      RETURN
      END


      FUNCTION DGAM(T)
      IMPLICIT REAL*8(A-H,O-Z)
C
C     CGAM SHOULD BE RETURNED AS RADIANS PER SECOND.
C
      CGAM=0.0C0
      RETURN
      END


      FUNCTION DDGAM(T)
      IMPLICIT REAL*8(A-H,O-Z)
C
C     DDGAM SHOULD BE RETURNED AS RADIANS PER SEC. * SEC.
C
      CDGAM=0.0D0
      RETURN
      END


      FUNCTION ALPHA(T)
      IMPLICIT REAL*8(A-H,O-Z)
C
C     ALPHA SHOULD BE RETURNED IN RADIANS.
C
      ALPHA=.1025800
      RETURN
      END


      FUNCTION DALPHA(T)
      IMPLICIT REAL*8(A-H,O-Z)
C
C     DALPHA SHOULD BE RETURNED IN RADIANS PER SECCND.
C
      DALPHA=0.0D0
      RETURN
      END


      FUNCTION W(T)
      IMPLICIT REAL*8(A-H,O-Z)
      W=2650.-58259.0*.000CCC304*T
      RETURN
      ENC


      FUNCTION DW(T)
      IMPLICIT REAL*8(A-H,O-Z)
      CW=-58259.0*.000000304
      RETURN
      ENC


      FUNCTION P(V,H)
      IMPLICIT REAL*8(A-H,O-Z)
C
C     P SHOULD BE RETURNED IN FT-LBS. PER SECOND.
C
      CALL POMAX(V,H,PMAX)
      P=PMAX
      RETURN
      END


      SUBROUTINE TRENOR(N,H,X,Y,DY,F,HMAX,ACCHLV,ACCDBL,MAXHLV,KNTHLV,IS
     1ET)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION YR(5),Y(5),Y2(5),Y3(5),Y4(5),YL(5),YLL(5),YLLL(5),YLLLL(
     15),CYR(5),DY(5),DY2(5),DY3(5),DY4(5),DYL(5),DYLL(5),DYLLL(5),DYLLL
     1L(5),P(5),F1(5),F2(5),F3(5)
      EXTERNAL F
      COMMON KEY,JKEY(5)
C
C     THIS INTEGRATION TECHNIQUE IS A RUNGE-KUTTA ADAMS-BASHFCRTH METHOD
C       WHICH HAS BEEN MODIFIED TO ACCOUNT FOR RELAXATION TYPE EQUATIONS
C
      CALL F(X,Y,DY,&26)
      IERR=0
      IF (ISET)  2,2,4
    2 CO 3 J=1,N
      I=JKEY(J)
      YLLLL(I)=Y(I)
      YLLL(I)=Y(I)
      YLL(I)=Y(I)
      YL(I)=Y(I)
      DYLLLL(I)=DY(I)
      DYLLL(I)=DY(I)
      DYLL(I)=DY(I)
    3 DYL(I)=DY(I)
      XLLLL=X
      XLLL=X
      XLL=X
      XL=X
      K=1
      ISET=1
      GO TO 9
    4 GO TO (5,6,7,8,15),K
    5 K=2
      CO TC 9
    6 K=3
      GO TO 9
```

```
    7 K=4
      GO TO 9
    8 K=5
      GO TO 15
    9 DO 10 J=1,N
      I=JKEY(J)
   10 Y2(I)=Y(I)+0.5D0*H*DY(I)
      CALL F(X+0.5D0*H,Y2,DY2,&26)
      DO 11 J=1,N
      I=JKEY(J)
   11 Y3(I)=Y(I)+0.5D0*H*DY2(I)
      CALL F(X+0.5D0*H,Y3,DY3,&26)
      DO 13 J=1,N
      I=JKEY(J)
      IF ((DY2(I)-DY(I)).EQ.0.0) GO TO 12
      P(I)=-2.0D0*(DY3(I)-DY2(I))/(DY2(I)-DY(I))/H
      IF (P(I).LE.0.0D0) GO TO 12
      F0=DEXP(-P(I)*H)
      F1(I)=-(F0-1.0D0)/P(I)/H
      F2(I)=-(F1(I)-1.0D0)/P(I)/H
      F3(I)=-(F2(I)-0.5D0)/P(I)/H
      Y4(I)=Y(I)+H*(2.0D0*DY3(I)*F2(I)+DY(I)*(F1(I)-2.0D0*F2(I))+DY2(I)*
     1P(I)*H*F2(I))
      GO TO 13
   12 F1(I)=1.0D0
      F2(I)=0.5D0
      F3(I)=1.666666666666667D-1
      P(I)=0.0D0
      Y4(I)=Y(I)+H*DY3(I)
   13 CONTINUE
      CALL F(X+H,Y4,DY4,&26)
      DO 14 J=1,N
      I=JKEY(J)
      B=-3.0D0*(DY(I)+P(I)*Y(I))+2.0D0*(DY2(I)+P(I)*Y2(I))+2.0D0*(DY3(I)
     1+P(I)*Y3(I))-(DY4(I)+P(I)*Y4(I))
      C=4.0D0*((DY(I)+P(I)*Y(I))-(DY2(I)+P(I)*Y2(I))-(DY3(I)+P(I)*Y3(I))
     1+(DY4(I)+P(I)*Y4(I)))
   14 YR(I)=Y(I)+H*(DY(I)*F1(I)+B*F2(I)+C*F3(I))
      CALL F(X+H,YR,DYR,&26)
      GO TO 28
C
   15 DO 16 J=1,N
      I=JKEY(J)
   16 P(I)=Y(I)+H*(55.0D0*DY(I)-59.0D0*DYL(I)+37.0D0*DYLL(I)-9.0D0*DYLLL
     1(I))/24.0D0
      KNT=1
   17 CALL F(X+H,P,DYR,&26)
      DO 21 J=1,N
      I=JKEY(J)
      IF ((P(I)-4.0D0*Y(I)+6.0D0*YL(I)-4.0D0*YLL(I)+YLLL(I)).EQ.0.0) GO
     1TO 18
      PP=-(DYR(I)-4.0D0*DY(I)+6.0D0*DYL(I)-4.0D0*DYLL(I)+DYLLL(I))/(P(I)
     1-4.0D0*Y(I)+6.0D0*YL(I)-4.0D0*YLL(I)+YLLL(I))
      IF (PP.LE.0.0D0) GO TO 18
      FF0=DEXP(-PP*H)
      FF1=-(FF0-1.0D0)/PP/H
      FF2=-(FF1-1.0D0)/PP/H
      FF3=-(FF2-0.5D0)/PP/H
      FF4=-(FF3-1.666666666666667D-1)/PP/H
      GO TO 19
   18 PP=0.0D0
      FF1=1.0D0
      FF2=0.5D0
      FF3=1.666666666666667D-1
      FF4=4.166666666666667D-2
   19 B=3.333333333333333D-1*(DYR(I)+PP*P(I))+0.5D0*(DY(I)+PP*Y(I))-(DYL
     1(I)+PP*YL(I))+1.666666666666667D-1*(DYLL(I)+PP*YLL(I))
      C=DYR(I)+PP*P(I)-2.0D0*(DY(I)+PP*Y(I))+DYL(I)+PP*YL(I)
      D=DYR(I)+PP*P(I)-3.0D0*(DY(I)+PP*Y(I))+3.0D0*(DYL(I)+PP*YL(I))-(DY
     1LL(I)+PP*YLL(I))
      IF (DABS(Y(I)).LT.1.D-20) GO TO 20
      YKEEP=H*(DY(I)*FF1+B*FF2+C*FF3+D*FF4)
      IF (DABS(YKEEP/Y(I)).GT.1.0) GO TO 26
   20 YR(I)=Y(I)+H*(DY(I)*FF1+B*FF2+C*FF3+D*FF4)
   21 CONTINUE
      CALL F(X+H,YR,DYR,&26)
      IF (KNT-2) 22,24,24
   22 DO 23 J=1,N
      I=JKEY(J)
   23 P(I)=YR(I)
      KNT=2
      GO TO 17
   24 IHLV=0
      IDBL=0
      DO 25 J=1,N
      I=JKEY(J)
      IF (YR(I).EQ.0.0D0)IDBL=IDBL+1
      IF (YR(I).EQ.0.0D0) GO TO 25
      ERROR=DABS((YR(I)-P(I))/YR(I))
      IF (ERROR.LT.ACCDBL)IDBL=IDBL+1
      IF (ERROR.GT.ACCHLV)IHLV=IHLV+1
   25 CONTINUE
      IF (IDBL.EQ.N)IERR=1
      IF (IHLV.NE.0)IERR=-1
      GO TO 27
   26 IERR=-1
   27 IF (IERR)  31,28,28
   28 DO 29 J=1,N
      I=JKEY(J)
      YLLLL(I)=YLLL(I)
      YLLL(I)=YLL(I)
      YLL(I)=YL(I)
      YL(I)=Y(I)
      Y(I)=YR(I)
      DYLLLL(I)=DYLLL(I)
      DYLLL(I)=DYLL(I)
      DYLL(I)=DYL(I)
      DYL(I)=DY(I)
   29 DY(I)=DYR(I)
      XLLLL=XLLL
      XLLL=XLL
      XLL=XL
      XL=X
      X=X+H
      IF (IERR.EQ.0) RETURN
C
C     DOUBLE INTERVAL
C
      IF (H.EQ.HMAX) RETURN
      IF ((H+H).LT.HMAX)HTMP=H+H
      IF ((H+H).GE.HMAX)HTMP=HMAX
      H=HTMP
      KNTHLV=KNTHLV-1
      ISET=0
      WRITE (3,30) X,H
   30 FORMAT (1X,'**** INTERVAL DOUBLED AT T = ',D11.4,2X,'SECONDS',5X,'
     1 NEW TDEL = ',D11.4,2X,'SECONDS  ****')
      RETURN
```

```
C
C     HALVE INTERVAL
C
   31 ISET=0
      DO 32 J=1,N
      I=JKEY(J)
   32 Y(I)=YLLLL(I)
      X=XLLLL
      H=H*0.5D0
      CALL F(X,Y,DY,&26)
      WRITE (3,33) X,H
   33 FORMAT (1X,'**** INTERVAL HALVED AT T = ',D11.4,2X,'SECONDS',5X,'
     1 NEW TDEL = ',D11.4,2X,'SECONDS  ****')
      KNTHLV=KNTHLV+1
      IF (KNTHLV.LT.MAXHLV) RETURN
      WRITE (3,34)
   34 FORMAT (/,1X,'VALUE OF MAXHLV WAS EXCEEDED, EXECUTION STOPPED',/)
      MAXHLV=-1
      RETURN
      END
```

Sample Output

POWER AVAILABLE VS. VELOCITY
REFERENCE ALTITUDE = 0.0 FEET

PA(FT-LBS/SEC)	VA(FT/SEC)
0.0	0.0
0.291500 05	0.273300 02
0.524700 05	0.546700 02
0.699600 05	0.820000 02
0.816200 05	0.109330 03
0.874500 05	0.136670 03
0.909480 05	0.164000 03
0.944460 05	0.191330 03
0.958650 05	0.218670 03
0.967780 05	0.246000 03
0.973610 05	0.273330 03
0.979440 05	0.300670 03
0.994700 05	0.328000 03
0.994700 05	0.355330 03
0.994700 05	0.382660 03

H1 = 10000.	V1 = 149.0	GAM1 = 0.0	ALPHA1 = 5.878	W1 = 2650.
P1 = 49.40	X1 = 0.0	C = 0.6000	TDEL = 0.5000	TMAX = 480.0
G = 32.20	S = 174.0	RHO = 0.2380D-02	CLA0 = 0.3090	CLA = 4.608
CD0 = 0.2690D-01	E = 0.9800	AR = 7.378	KEY = 3	IPRINT = 10
MAXHLV = 2	ACCHLV = 0.1000D-02	ACCDBL = 0.2000D-04	HMAX = 60.00	VMIN = 35.00
WEMPTY = 2150.	EX = 2.000			

TIME (MIN)	ALTITUDE (FT)	VELOCITY (FT/SEC)	GAMMA (DEG)	ALPHA (DEG)	CL	CD	WEIGHT (POUNDS)	POWER (HP)	RANGE (MILES)
0.0	10000.	149.0	0.0	5.877	0.7817	0.5380D-01	2650.	49.40	0.0
**** INTERVAL DOUBLED AT T =		0.2500D 01	SECONDS	NEW TDEL =	0.1000D 01	SECONDS ****			
**** INTERVAL DOUBLED AT T =		0.7500D 01	SECONDS	NEW TDEL =	0.2000D 01	SECONDS ****			
0.1250	10000.	149.0	0.0	5.877	0.7817	0.5380D-01	2650.	49.40	0.2116
**** INTERVAL DOUBLED AT T =		0.1750D 02	SECONDS	NEW TDEL =	0.4000D 01	SECONDS ****			
**** INTERVAL DOUBLED AT T =		0.3750D 02	SECONDS	NEW TDEL =	0.8000D 01	SECONDS ****			
0.6250	10000.	149.0	0.0	5.877	0.7817	0.5380D-01	2650.	49.39	1.058
**** INTERVAL DOUBLED AT T =		0.7750D 02	SECONDS	NEW TDEL =	0.1600D 02	SECONDS ****			
**** INTERVAL DOUBLED AT T =		0.1575D 03	SECONDS	NEW TDEL =	0.3200D 02	SECONDS ****			
2.625	10000.	149.0	0.0	5.877	0.7817	0.5380D-01	2649.	49.37	4.444
**** INTERVAL DOUBLED AT T =		0.3175D 03	SECONDS	NEW TDEL =	0.6000D 02	SECONDS ****			
10.29	10000.	148.9	0.0	5.877	0.7817	0.5380D-01	2645.	49.26	17.42
20.29	10000.	148.7	0.0	5.877	0.7817	0.5380D-01	2640.	49.12	34.33
30.29	10000.	148.6	0.0	5.877	0.7817	0.5380D-01	2635.	48.99	51.22
40.29	10000.	148.4	0.0	5.877	0.7817	0.5380D-01	2630.	48.85	68.09
50.29	10000.	148.3	0.0	5.877	0.7817	0.5380D-01	2625.	48.71	84.95
60.29	10000.	148.2	0.0	5.877	0.7817	0.5380D-01	2620.	48.58	101.8
70.29	10000.	148.0	0.0	5.877	0.7817	0.5380D-01	2616.	48.44	118.6
80.29	10000.	147.9	0.0	5.877	0.7817	0.5380D-01	2611.	48.31	135.4
90.29	10000.	147.8	0.0	5.877	0.7817	0.5380D-01	2606.	48.18	152.2
100.3	10000.	147.6	0.0	5.877	0.7817	0.5380D-01	2601.	48.04	169.0
110.3	10000.	147.5	0.0	5.877	0.7817	0.5380D-01	2596.	47.91	185.8
120.3	10000.	147.3	0.0	5.877	0.7817	0.5380D-01	2592.	47.78	202.5
130.3	10000.	147.2	0.0	5.877	0.7817	0.5380D-01	2587.	47.65	219.3
140.3	10000.	147.1	0.0	5.877	0.7817	0.5380D-01	2582.	47.51	236.0
150.3	10000.	146.9	0.0	5.877	0.7817	0.5380D-01	2577.	47.38	252.7
160.3	10000.	146.8	0.0	5.877	0.7817	0.5380D-01	2573.	47.25	269.4
170.3	10000.	146.7	0.0	5.877	0.7817	0.5380D-01	2568.	47.12	286.1
180.3	10000.	146.5	0.0	5.877	0.7817	0.5380D-01	2563.	46.99	302.7
190.3	10000.	146.4	0.0	5.877	0.7817	0.5380D-01	2558.	46.87	319.4
200.3	10000.	146.3	0.0	5.877	0.7817	0.5380D-01	2554.	46.74	336.0
210.3	10000.	146.1	0.0	5.877	0.7817	0.5380D-01	2549.	46.61	352.6
220.3	10000.	146.0	0.0	5.877	0.7817	0.5380D-01	2544.	46.48	369.2
230.3	10000.	145.9	0.0	5.877	0.7817	0.5380D-01	2540.	46.35	385.8
240.3	10000.	145.7	0.0	5.877	0.7817	0.5380D-01	2535.	46.23	402.4
250.3	10000.	145.6	0.0	5.877	0.7817	0.5380D-01	2531.	46.10	418.9
260.3	10000.	145.5	0.0	5.877	0.7817	0.5380D-01	2526.	45.98	435.5
270.3	10000.	145.3	0.0	5.877	0.7817	0.5380D-01	2521.	45.85	452.0
280.3	10000.	145.2	0.0	5.877	0.7817	0.5380D-01	2517.	45.73	468.5
290.3	10000.	145.1	0.0	5.877	0.7817	0.5380D-01	2512.	45.60	485.0
300.3	10000.	144.9	0.0	5.877	0.7817	0.5380D-01	2508.	45.48	501.5
310.3	10000.	144.8	0.0	5.877	0.7817	0.5380D-01	2503.	45.35	517.9
320.3	10000.	144.7	0.0	5.877	0.7817	0.5380D-01	2499.	45.23	534.4
330.3	10000.	144.5	0.0	5.877	0.7817	0.5380D-01	2494.	45.11	550.8
340.3	10000.	144.4	0.0	5.877	0.7817	0.5380D-01	2490.	44.99	567.2
350.3	10000.	144.3	0.0	5.877	0.7817	0.5380D-01	2485.	44.86	583.6
360.3	10000.	144.2	0.0	5.877	0.7817	0.5380D-01	2481.	44.74	600.0
370.3	10000.	144.0	0.0	5.877	0.7817	0.5380D-01	2476.	44.62	616.4
380.3	10000.	143.9	0.0	5.877	0.7817	0.5380D-01	2472.	44.50	632.8
390.3	10000.	143.8	0.0	5.877	0.7817	0.5380D-01	2467.	44.38	649.1
400.3	10000.	143.6	0.0	5.877	0.7817	0.5380D-01	2463.	44.26	665.4
410.3	10000.	143.5	0.0	5.877	0.7817	0.5380D-01	2458.	44.14	681.7
420.3	10000.	143.4	0.0	5.877	0.7817	0.5380D-01	2454.	44.02	698.0
430.3	10000.	143.3	0.0	5.877	0.7817	0.5380D-01	2450.	43.91	714.3
440.3	10000.	143.1	0.0	5.877	0.7817	0.5380D-01	2445.	43.79	730.6
450.3	10000.	143.0	0.0	5.877	0.7817	0.5380D-01	2441.	43.67	746.9
460.3	10000.	142.9	0.0	5.877	0.7817	0.5380D-01	2436.	43.55	763.1
470.3	10000.	142.7	0.0	5.877	0.7817	0.5380D-01	2432.	43.44	779.3
480.3	10000.	142.6	0.0	5.877	0.7817	0.5380D-01	2428.	43.32	795.5

H1 = 0.0	V1 = 120.0	GAM1 = 1.500	ALPHA1 = 7.420	W1 = 2700.
P1 = 153.5	X1 = 0.0	C = 0.6000	TDEL = 0.5000	TMAX = 30.00
G = 32.20	S = 174.0	RHO = 0.2380D-02	CLA0 = 0.3090	CLA = 4.608
CD0 = 0.2987D-01	E = 0.5930	AR = 7.378	KEY = 12	IPRINT = 5
MAXHLV = 2	ACCHLV = 0.1000D-02	ACCDBL = 0.2000D-04	HMAX = 30.00	VMIN = 35.00
WEMPTY = 2150.	EX = 2.991			

TIME (MIN)	ALTITUDE (FT)	VELOCITY (FT/SEC)	GAMMA (DEG)	ALPHA (DEG)	CL	CD	WEIGHT (PCUNCS)	POWER (HP)	RANGE (MILES)
0.0	0.0	120.0	1.500	7.416	0.9054	0.8392D-01	2700.	153.5	0.0
**** INTERVAL DOUBLED AT T = 0.2500D 01 SECONDS				NEW TDEL = 0.1000D 01 SECONDS ****					
0.4167D-01	8.223	131.3	1.500	5.568	0.7568	0.6149D-01	2700.	157.4	0.5948D-01
**** INTERVAL DOUBLED AT T = 0.7500D 01 SECONDS				NEW TDEL = 0.2000D 01 SECONDS ****					
0.1250	26.78	151.7	1.500	3.208	0.5670	0.4320D-01	2700.	162.4	0.1937
**** INTERVAL DOUBLED AT T = 0.1750D 02 SECONDS				NEW TDEL = 0.4000D 01 SECONDS ****					
0.2917	70.76	182.2	1.500	1.049	0.3934	0.3434D-01	2700.	169.5	0.5118
**** INTERVAL DOUBLED AT T = 0.3750D 02 SECONDS				NEW TDEL = 0.8000D 01 SECONDS ****					
0.6250	175.7	213.8	1.500	-0.2776	0.2867	0.3160D-01	2699.	172.9	1.271
**** INTERVAL DOUBLED AT T = 0.7750D 02 SECONDS				NEW TDEL = 0.1600D 02 SECONDS ****					
1.292	410.3	229.3	1.500	-0.7230	0.2509	0.3103D-01	2698.	172.5	2.968
**** INTERVAL DOUBLED AT T = 0.1575D 03 SECONDS				NEW TDEL = 0.3000D 02 SECONDS ****					
2.625	893.5	231.0	1.500	-0.7283	0.2504	0.3103D-01	2696.	169.6	6.463
5.125	1798.	229.8	1.500	-0.6170	0.2594	0.3116D-01	2691.	164.2	13.01
7.625	2698.	228.6	1.500	-0.4982	0.2689	0.3130D-01	2687.	158.9	19.52
10.13	3593.	227.3	1.500	-0.3726	0.2790	0.3147D-01	2683.	153.7	25.99
12.63	4483.	225.9	1.500	-0.2394	0.2897	0.3166D-01	2680.	148.7	32.42
15.13	5367.	224.4	1.500	-0.9787D-01	0.3011	0.3188D-01	2676.	143.8	38.82
17.63	6245.	222.8	1.500	0.5278D-01	0.3132	0.3213D-01	2672.	139.0	45.17
20.13	7116.	221.1	1.500	0.2136	0.3262	0.3242D-01	2669.	134.3	51.47
22.63	7981.	219.4	1.500	0.3859	0.3400	0.3276D-01	2666.	129.8	57.73
25.13	8839.	217.5	1.500	0.5710	0.3549	0.3315D-01	2663.	125.4	63.93
27.63	9688.	215.4	1.500	0.7710	0.3710	0.3362D-01	2659.	121.2	70.08
30.13	0.1053D 05	213.2	1.500	0.9881	0.3885	0.3417D-01	2656.	117.0	76.16

APPENDIX E - Lift-Drag Curve Fitting Program

User Instructions

Using a Least-Square-Distance curve fit procedure (Ref. 26) this program yields a general drag polar of the form

$$C_D = k_1 + K_2C_L^2 + k_3C_L^{k_4}$$

where drag coefficient versus lift coefficient data is supplied. The program is written in FORTRAN IV and is designed to run in double precision on an IBM 370-165 computer with an average execution time of three to five seconds per curve fit investigated. Since this program is designed to be used in conjunction with the programs in Appendices C and D, the user has the option of four types of the general drag polar:

(1) $C_D = k_1 + k_2C_L^2 + k_3C_L^{k_4}$ where all four coefficients k_1, k_2, k_3, and k_4 are varied in the fitting process,

(2) $C_D = C_{D_O} + k_2C_L^2 + k_3C_L^{k_4}$ where C_{D_O} is specified by the user and k_2, k_3, and k_4 are varied,

(3) $C_D = k_1 + k_3C_L^{k_4}$ where k_1, k_3, and k_4 are varied.

(4) $C_D = C_{D_O} + k_3C_L^{k_4}$ where k_3 and k_4 are varied and C_{D_O} is specified by the user.

For options (2) and (4) the first specified data point must be the zero-lift drag coefficient. Note that the functional form of the general polar prohibits the use of negative values of C_L.

The program requires the specification of the following input data:

(1) The number N of drag coefficient versus lift coefficient data points, the control parameter IKEY which specifies which of the four types of the general drag polar is to be used as the fitting functions,

IKEY = 1 $\quad C_D = k_1 + k_2C_L^2 + k_3C_L^{k_4}$

IKEY = 2 $\quad C_D = C_{D_O} + k_2C_L^2 + k_3C_L^{k_4}$

IKEY = 3 $\quad C_D = k_1 + k_3C_L^{k_4}$

IKEY = 4 $\quad C_D = C_{D_O} + k_3C_L^{k_4}$,

(2) The N data points of drag coefficient C_D versus lift coefficient C_L, one data point per card with C_D specified first in ascending order based on the magnitude of C_L.

Statements (1) and (2) above represent a complete set of data for one fitting process. Table 1 gives the format specification of the input data. Upon completion of the calculations with a complete set of data the program returns to the statement where N and IKEY are read. This gives the user the option of fitting another data set if desired. If N is positive when N and IKEY are read, a curve fit is performed on the new set of data. If N is zero the program terminates. Thus, the final data in the program must specify N as zero.

Columns 1–10	Columns 11–20	Columns 21–40
N	IKEY	
I10	I10	
CD(1)		CL(1)
D20.13		D20.13
.		.
.		.
.		.
CD(N)		CL(N)
D20.13		D20.13

Table E-1. Data format for curve fit program.

Listing

```
C
C
C     PROGRAM TO OBTAIN A GENERAL DRAG POLAR FROM DRAG COEFFICIENT
C     VS. LIFT COEFFICIENT DATA BY A LEAST-SQUARE-DISTANCE CURVE FIT
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION CL(30),CD(30),AL(4)
      COMMON CD0,IKEY
      DATA ERR/5.0D-03/
C
C
C     INPUT DATA
C
C        N      - NUMBER OF DATA POINTS
C        IKEY   - CONTROL PARAMETER THAT SPECIFIES TYPE OF DRAG POLAR
C                 IKEY = 1  -> CD = AL(1) + AL(2)*CL**2 + AL(3)*CL**AL(4)
C                 IKEY = 2  -> CD = CD0 + AL(1)*CL**2 + AL(2)*CL**AL(3)
C                 IKEY = 3  -> CD = AL(1) + AL(2)*CL**AL(3)
C                 IKEY = 4  -> CD = CD0 + AL(1)*CL**AL(2)
C                 IF IKEY = 2 OR 4 THEN CD(1) MUST BE CD0, THE ZERO-
C                 LIFT DRAG COEFFICIENT, AND CL(1) = 0.0
C        CL     - LIFT COEFFICIENTS IN ASCENDING ORDER.
C                 CL(1) IS THE SMALLEST, AND CL(N) IS THE LARGEST
C                 NONE OF THE CL VALUES MAY BE NEGATIVE
C        CD     - DRAG COEFFICIENTS
C        AL     - COEFFICIENTS OF THE DRAG POLAR THAT ARE TO BE FITTED
C                 BY THE LEAST-SQUARE-DISTANCE TECHNIQUE
C
    1 READ(1,100) N,IKEY
  100 FORMAT(2I10)
      IF(N.EQ.0) CALL EXIT
      READ(1,101) (CD(I),CL(I),I=1,N)
  101 FORMAT(2D20.13)
      WRITE(3,200) (CD(I),CL(I),I=1,N)
  200 FORMAT(1H1,57X,'DATA TO BE FITTED'//1X,57X,'CD',13X,'CL'/(1X,51X,2
     $(D12.5,3X)))
      WRITE(3,201)
  201 FORMAT(///)
C
C     GENERATE INITIAL GUESSES FOR THE AL(I)
C
      GO TO (2,2,5,5),IKEY
    2 M = N/2
      CDNM = CD(N) - CD(M)
      CDM1 = CD(M) - CD(1)
      CLNMSQ = CL(N)**2 - CL(M)**2
      CLM1SQ = CL(M)**2 - CL(1)**2
      CLNMCU = CL(N)**3 - CL(M)**3
      CLM1CU = CL(M)**3 - CL(1)**3
      COEF3 = (CDNM-CLNMSQ*CDM1/CLM1SQ)/(CLNMCU -CLNMSQ*CLM1CU/CLM1SQ)
      COEF2 = (CDNM - COEF3*CLNMCU)/CLNMSQ
      IF(IKEY-1) 3,3,4
    3 AL(4) = 3.0D0
      AL(3) = COEF3
      AL(2) = COEF2
      AL(1) = CD(1) - AL(2)*CL(1)**2 - AL(3)*CL(1)**3
      M = 4
      GO TO 8
    4 AL(3) = 3.0D0
      AL(2) = COEF3
      AL(1) = COEF2
      CD0 = CD(1)
      M = 3
      GO TO 8
    5 COEF3 = (CD(N)-CD(1))/(CL(N)**3-CL(1)**3)
      IF(IKEY-3) 6,6,7
    6 AL(2) = 3.0D0
      AL(2) = COEF3
      AL(1) = CD(1) - AL(2)*CL(1)**3
      M = 3
      GO TO 8
    7 AL(2) = 3.0D0
      AL(1) = COEF3
      CD0 = CD(1)
      M = 2
C
C     OBTAIN LEAST-SQUARE-DISTANCE FIT OF DATA
C
C        M      - NUMBER OF COEFFICIENTS TO BE FITTED
C        ERR    - RELATIVE CONVERGENCE CRITERIA ON RMS
C        RMS    - ROOT-MEAN-SQUARE DEVIATION BASED ON THE PERPEN-
C                 DICULAR DISTANCES, D(I), FROM THE DATA POINTS
C                 TO THE FITTED CURVE
C                 RMS = SUM(D(I)**2,I=1,N)/N
C
    8 CALL LSD(CL,CD,N,AL,M,ERR,RMS)
      GO TO (9,10,11,12),IKEY
    9 WRITE(3,202) RMS,(AL(I),I=1,4)
  202 FORMAT(1X///1X,57X,'RMS =',D12.5/1X,34X,'CD =',D12.5,' + ',D12.5,'
     $*CL**2 + ',D12.5,'*CL**',D12.5/////)
      GO TO 1
   10 WRITE(3,203) RMS,CD0,(AL(I),I=1,3)
  203 FORMAT(1X///1X,57X,'RMS =',D12.5/1X,34X,'CD =',D12.5,' + ',D12.5,'
     $*CL**2 + ',D12.5,'*CL**',D12.5/////)
      GO TO 1
   11 WRITE(3,204) RMS,(AL(I),I=1,3)
  204 FORMAT(1X///1X,57X,'RMS =',D12.5/1X,24X,'CD =',D12.5,' + ',D12.5,'
     $*CL**',D12.5/////)
      GO TO 1
   12 WRITE(3,205) RMS,CD0,(AL(I),I=1,2)
  205 FORMAT(1X///1X,57X,'RMS =',D12.5/1X,24X,'CD =',D12.5,' + ',D12.5,'
     $*CL**',D12.5/////)
      GO TO 1
      END
```

```
      SUBROUTINE LSD(X,Y,N,AL,M,ERR,RMS)
C
C
C
C     LEAST SQUARES DISTANCE CURVE FITTING SUBROUTINE
C
C     THIS SUBROUTINE PERFORMS A LEAST-SQUARE-DISTANCE CURVE FIT
C     OF A PRESCRIBED FUNCTION Y = F(X) TO THE DATA SET (X(I),Y(I))
C
C        X      - INDEPENDENT VARIABLE OF DATA SET
C        Y      - DEPENDENT VARIABLE OF DATA SET
C        N      - NUMBER OF DATA POINTS
C        AL     - COEFFICIENTS OF PRESCRIBED FUNCTION THAT
C                 ARE FITTED BY THE LEAST-SQUARE-DISTANCE METHOD
C        M      - NUMBER OF COEFFICIENTS TO BE FITTED
C        ERR    - RELATIVE CONVERGENCE CRITERIA ON RMS
C        RMS    - ROOT-MEAN-SUARE DEVIATION BASED ON THE PERPEN-
C                 DICULAR DISTANCES, D(I), FROM THE DATA POINTS
C                 TO THE FITTED CURVE
C
C     THIS SUBROUTINE USES THE FOLLOWING THREE SUBROUTINES:
C
C        FNCTNA(AL,X,Y,DYDX)      - RETURNS THE VALUE OF THE FITTED
C                                   FUNCTION & ITS X-DERIVATIVE AS Y
C                                   & DYDX WHEN GIVEN AN ARGUEMENT X
C                                   AND COEFFICIENTS AL(M)
C        FNCTNB(AL,M,X,Y,DYDAL)   - RETURNS THE VALUE OF THE FITTED
C                                   FUNCTION & ITS PARTIAL DERIVATIVES
C                                   WRT THE COEFFICIENTS AL(M) IN THE
```

```
C                          ARRAYS Y(N) & DYDAL(M,N) WHEN GIVEN
C                          THE ARRAY ARGUEMENTS X(N) AND COEF-
C                          FICIENTS AL(M)
C      SIMSOL(A,B,M)     - SOLVES THE LINEAR SIMULTANEOUS ECUA-
C                          TION AX=B IN M UNKNOWNS, AND RETURNS
C                          THE SOLUTIONS IN ARRAY B(M)
C
C     THIS SUBROUTINE IS A MODIFICATION OF THE GENERAL LEAST-SQUARE-
C     DISTANCE CURVE FITTING PROGRAM GIVEN IN THE FOLLOWING
C     REFERENCE:
C
C       A LEAST-SQUARE-DISTANCE CURVE-FITTING TECHNIQUE
C       JOHN Q. HOWELL, NASA TN D-6374, JULY 1971
C
C
C     BECAUSE OF THE EXTREME NON-LINEARITY OF THE FUNCTION BEING
C     FITTED, I.E. A DRAG POLAR WITH A VARIABLE EXPONENT ON CL AS
C     ONE OF THE COEFFICIENTS TO BE FITTED, IT IS POSSIBLE FOR
C     THIS SUBROUTINE TO CONVERGE TO SEVERAL DIFFERENT FITS OF
C     THE DATA.  THIS WILL HAPPEN IF THE ERROR PARAMETER ERR IS
C     MADE TOO SMALL.  THE SUBROUTINE CONVERGES TO A FIT OF THE
C     DATA, BUT WITH A RMS GREATER THAN THAT ALLOWED BY THE ERROR
C     PARAMETER.  THE SUBROUTINE CONTINUES TO ITERATE ON THE COEF-
C     FICIENTS IN AN ATTEMPT TO REDUCE THE RMS.  THIS PROCESS BE-
C     COMES UNSTABLE, AND THE COEFFICIENTS DIVERGE AWAY FROM A
C     GOOD FIT.  AS THE ITERATION CONTINUES, THEY THEN CONVERGE
C     BACK TO A NEW AND DIFFERENT FIT OF THE DATA.  THIS NEW FIT
C     GENERALLY HAS A LARGER EXPONENT ON THE EXTRA CL TERM OF THE
C     DRAG POLAR THAN THE PREVIOUS FIT.  IN ORDER TO ALLOW THE
C     USER TO PICK THE BEST FIT OUT OF THESE POSSIBLE SEVERAL CON-
C     VERGENT FITS, THE SUBROUTINE PRINTS OUT THE CURRENT VALUES
C     OF RMS AND THE COEFFICIENTS AL(M) AFTER EVERY FIVE ITERATIONS.
C
      SUBROUTINE LSD(X,Y,N,AL,M,ERR,RMS)
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(4,4),AL(4),B(4),DIS(30),DYDAL(4,30),W(30),X(30),XX(30)
     $,Y(30),YY(30)
      RMSOLD = 0.0D0
      IPRINT = 1
C
C     START ITERATION
C
      DO 23 ITER=1,100
C
C     FIND CLOSEST POINTS ON CURVE
C
      IFAIL = 0
      DO 12 I=1,N
      W(I) = 1.0D0
C
C     OBTAIN INITIAL GUESS AT X-COORDINATE OF CLOSEST POINT
C     (XX(I),YY(I)) ON CURVE TO DATA POINT (X(I),Y(I)) FROM THE
C     INTERSECTION OF A PERPENDICULAR FROM THE DATA POINT WITH
C     THE TANGENT TO THE CURVE AT THE POINT (X(I),F(X(I))) WHERE
C     THE CURVE EQUATION IS Y = F(X)
C
      CALL FNCTNA(AL,X(I),F,DFDX)
      XX(I) = X(I) - (F - Y(I))*DFDX/(1.0D0 + DFDX*DFDX)
C
C     SEARCH ABOUT THIS POINT XX(I) FOR A SIGN CHANGE IN THE
C     FUNCTION  (F(X) - Y(I))*DFDX + (X - X(I))
C
      IF(X(I).NE.0.0D0) GO TO 1
      YY(I) = F
      XX(I) = X(I)
      GO TO 11
    1 IF(I.NE.1) GO TO 2
      DXUPR = 0.01D0*(X(I+1) - X(I))
      DXLWR = 0.01D0*X(I)
      GO TO 3
    2 DXLWR = 0.01D0*(X(I) - X(I-1))
      DXUPR = DXLWR
      IF(I.NE.N) DXUPR = 0.01D0*(X(I+1) - X(I))
    3 XLWR = XX(I)
      XUPR = XLWR
      DO 5 J=1,100
      CALL FNCTNA(AL,XLWR,FLWR,DFLWR)
      CALL FNCTNA(AL,XUPR,FUPR,DFUPR)
      FUNLWR = (FLWR - Y(I))*DFLWR + (XLWR - X(I))
      FUNUPR = (FUPR - Y(I))*DFUPR + (XUPR - X(I))
      IF(FUNLWR*FUNUPR) 6,4,4
    4 XLWR = XLWR - DXLWR
      XUPR = XUPR + DXUPR
      IF(XLWR.LT.0.0D0) XLWR = 0.0D0
    5 CONTINUE
      WRITE(3,100) I,X(I),Y(I)
  100 FORMAT(1X,10('*'),' SHORTEST DISTANCE NOT FOUND TO DATA POINT #',
     $I3,3X,'X = ',D12.5,3X,'Y = ',D12.5)
      DIS(I) = 0.0D0
      IFAIL = IFAIL + 1
      W(I) = 0.0D0
      GO TO 12
C
C     SIGN CHANGE HAS BEEN FOUND.  FIND ROOT BY METHOD OF FALSE
C     POSITION.
C
    6 XOLD = 0.0D0
      DO 9 J=1,100
      XROOT = (XLWR*FUNUPR - XUPR*FUNLWR)/(FUNUPR - FUNLWR)
      IF(DABS(1.0D0-XOLD/XROOT).LT.1.0D-03) GO TO 10
      XOLD = XROOT
      CALL FNCTNA(AL,XROOT,FROOT,DFROOT)
      FUNRT = (FROOT - Y(I))*DFROOT + (XROOT - X(I))
      IF(FUNLWR*FUNRT) 7,10,8
    7 FUNUPR = FUNRT
      XUPR = XROOT
      GO TO 9
    8 FUNLWR = FUNRT
      XLWR = XROOT
    9 CONTINUE
      WRITE(3,100) I,X(I),Y(I)
      DIS(I) = 0.0D0
      IFAIL = IFAIL + 1
      W(I) = 0.0D0
      GO TO 12
C
C     CLOSEST POINT FOUND, NOW FIND DISTANCE
C
   10 XX(I) = XROOT
      YY(I) = FROOT
   11 DX = XX(I) - X(I)
      DY = YY(I) - Y(I)
      DIS(I) = DSQRT(DX*DX + DY*DY)
   12 CONTINUE
C
C     COMPLETE SET OF CLOSEST POINTS AND DISTANCES FOUND
C     NOW FIND NEW SET OF PARAMETERS
C
C     FORM ROOT-MEAN-SQUARE DEVIATION AND CHECK FOR CONVERGENCE
C
      SUMDIS = 0.0D0
      DO 13 I=1,N
   13 SUMDIS = SUMDIS + DIS(I)*DIS(I)
      RMS = DSQRT(SUMDIS/(N-IFAIL))
      IF(DABS(RMS-RMSOLD).LT.ERR*RMS) RETURN
C
C     EVERY FIVE ITERATIONS PRINT CURRENT VALUES OF RMS AND THE
C     COEFFICIENTS AL(M)
C
      IF(ITER.EQ.IPRINT) WRITE(3,150) RMS,(AL(I),I=1,M)
  150 FORMAT(1X,'RMS =',D12.5/1X,'AL(I) =',4(G12.5,3X))
      IF(ITER.EQ.IPRINT) IPRINT = IPRINT + 5
C
C     GENERATE MATRIX COEFFICIENTS OF LINEAR SIMULTANEOUS EQUATIONS
C
      CALL FNCTNB(AL,N,XX,YY,DYDAL)
```

```
      DO 16 K=1,M
      B(K) = 0.0D0
      DO 16 I=1,N
      B(K) = B(K) + (Y(I) - YY(I))*DYDAL(K,I)*W(I)
      IF(W(I).EQ.0.0D0) GO TO 15
      IF(DIS(I).EQ.0.0D0) GO TO 14
      DYDAL(K,I) = (Y(I) - YY(I))*DYDAL(K,I)/DIS(I)
      GO TO 16
   14 CALL FNCTNA(AL,XX(I),F,DF)
      IF(DF.EQ.0.0D0) GO TO 15
      DYDAL(K,I) = DSQRT(1.0D0/(1.0D0 + 1.0D0/DF/DF))*DYDAL(K,I)
      GO TO 16
   15 DYDAL(K,I) = 0.0D0
   16 CONTINUE
      DO 19 K=1,M
      DO 18 J=K,M
      A(K,J) = 0.0D0
      DO 17 I=1,N
   17 A(K,J) = A(K,J) + DYDAL(K,I)*DYDAL(J,I)
   18 A(J,K) = A(K,J)
   19 CONTINUE
C
C     INCREASE STABILITY WITH DAMPING TECHNIQUE
C
      T = 0.0D0
      DO 20 I=1,M
   20 T = T + B(I)*B(I)
      WW = 0.5D0*SUMDIS/T
      DO 21 I=1,M
   21 A(I,I) = A(I,I) + 0.5D0/WW
C
C     SOLVE THE LINEAR SIMULTANEOUS EQUATION FOR CORRECTIONS TO
C     THE AL(I) AND GENERATE NEW AL(I)
C
      CALL SIMSOL(A,B,M)
      DO 22 I=1,M
   22 AL(I) = AL(I) + B(I)
   23 CONTINUE
      WRITE(3,200) ERR
  200 FORMAT(1X,10('*'),' AFTER 100 ITERATIONS (1.0 - RMS/RMSOLD), THE R
     $ELATIVE ERROR BETWEEN'/1X,11X,'TWO SUCCESSIVE VALUES OF RMS, STILL
     $ EXCEEDS THE SPECIFIED ERROR PARAMETER =',D10.3)
      RETURN
      END


      SUBROUTINE FNCTNA(AL,X,Y,DYDX)
C
C
C     THIS SUBROUTINE CALCULATES THE VALUE OF THE FUNCTION BEING
C     FITTED AND ITS X-DERIVATIVE AT THE POINT X
C
C        AL     - COEFFICIENTS OF THE FUNCTION THAT ARE BEING FITTED
C        X      - ARGUEMENT AT WHICH THE FUNCTION AND ITS X-DERIV-
C                 ATIVE ARE EVALUATED
C        Y      - VALUE OF FUNCTION AT X
C        DYDX   - DERIVATIVE OF FUNCTION AT X
C        IKEY   - CONTROL PARAMETER THAT DESIGNATES WHICH FUNCTIONAL
C                 FORM IS BEING FITTED
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION AL(4)
      COMMON CDO,IKEY
      IF(X.EQ.0.0D0) GO TO 5
      GO TO (1,2,3,4),IKEY
C
C     Y = AL(1) +AL(2)*X**2 + AL(3)*X**AL(4)
C
    1 Y = AL(1) + AL(2)*X**2 + AL(3)*X**AL(4)
      DYDX = 2.0D0*AL(2)*X + AL(3)*AL(4)*X**(AL(4)-1.0D0)
      RETURN
C
C     Y = CDO + AL(1)*X**2 + AL(2)*X**AL(3)
C
    2 Y = CDO + AL(1)*X**2 + AL(2)*X**AL(3)
      DYDX = 2.0D0*AL(1)*X + AL(2)*AL(3)*X**(AL(3)-1.0D0)
      RETURN
C
C     Y = AL(1) + AL(2)*X**AL(3)
C
    3 Y = AL(1) + AL(2)*X**AL(3)
      DYDX = AL(2)*AL(3)*X**(AL(3)-1.0D0)
      RETURN
C
C     Y = CDO + AL(1)*X**AL(2)
C
    4 Y = CDO + AL(1)*X**AL(2)
      DYDX = AL(1)*AL(2)*X**(AL(2)-1.0D0)
      RETURN
C
C
    5 Y = AL(1)
      IF(IKEY.EQ.2.OR.IKEY.EQ.4) Y = CDO
      DYDX = 0.0D0
      RETURN
      END


      SUBROUTINE FNCTNB(AL,N,X,Y,DYDAL)
C
C
C     THIS SUBROUTINE CALCULATES THE VALUE OF THE FITTED FUNCTION
C     AND ITS PARTIAL DERIVATIVES WITH RESPECT TO EACH OF THE
C     COEFFICIENTS AL FOR EACH POINT X(I)
C
C        AL     - COEFFICIENTS OF THE FUNCTION THAT ARE BEING FITTED
C        N      - NUMBER OF ARGUEMENT POINTS X(I)
C        X      - ARRAY CONTAINING ARGUEMENT POINTS
C        Y      - ARRAY RETURNING FUNCTION VALUES
C        DYDAL  - ARRAY RETURNING PARTIAL DERIVATIVES OF FUNCTION
C                 WITH RESPECT TO EACH COEFFICIENT AL
C        IKEY   - CONTROL PARAMETER THAT DESIGNATES WHICH FUNCTIONAL
C                 FORM IS BEING FITTED
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION AL(4),DYDAL(4,30),X(30),Y(30)
      COMMON CDO,IKEY
      GO TO (1,4,7,10),IKEY
C
C     Y(I) = AL(1)*X(I)**2 + AL(3)*X(I)**AL(4)
C
    1 DO 3 I=1,N
      XX = X(I)
      DYDAL(1,I) = 1.0D0
      DYDAL(2,I) = XX*XX
      IF(XX.EQ.0.0D0) GO TO 2
```

```
      DYDAL(3,I) = XX**AL(4)
      DYDAL(4,I) = AL(3)*XX**AL(4)*DLOG(XX)
      Y(I) = AL(1) +*AL(2)*XX**2 + AL(3)*XX**AL(4)
      GO TO 3
    2 DYDAL(3,I) = 0.0D0
      DYDAL(4,I) = 0.0D0
      Y(I) = AL(1)
    3 CONTINUE
      RETURN
C
C     Y(I) = C00 + AL(1)*X(I)**2 + AL(2)*X(I)**AL(3)
C
    4 DO 6 I=1,N
      XX = X(I)
      DYDAL(1,I) = XX*XX
      IF(XX.EQ.0.0D0) GO TO 5
      DYDAL(2,I) = XX**AL(3)
      DYDAL(3,I) = AL(2)*XX**AL(3)*DLOG(XX)
      Y(I) = C00 + AL(1)*XX**2 + AL(2)*XX**AL(3)
      GO TO 6
    5 DYDAL(2,I) = 0.0D0
      DYDAL(3,I) = C.0D0
      Y(I) = C00
    6 CONTINUE
      RETURN
C
C     Y(I) = AL(1) + AL(2)*X(I)**AL(3)
C
    7 DO 9 I=1,N
      XX = X(I)
      DYDAL(1,I) = 1.0C0
      IF(XX.EQ.0.0D0) GO TO 8
      DYDAL(2,I) = XX**AL(3)
      DYDAL(3,I) = AL(2)*XX**AL(3)*DLOG(XX)
      Y(I) = AL(1) + AL(2)*XX**AL(3)
      GO TO 9
    8 DYDAL(2,I) = 0.0D0
      DYDAL(3,I) = 0.0D0
      Y(I) = AL(1)
    9 CONTINUE
      RETURN
C
C     Y(I) = C00 + AL(1)*X(I)**AL(2)
C
   10 DO 12 I=1,N
      XX = X(I)
      IF(XX.EQ.0.0D0) GO TO 11
      DYDAL(1,I) = XX**AL(2)
      DYDAL(2,I) = AL(1)*XX**AL(2)*DLOG(XX)
      Y(I) = C00 + AL(1)*XX**AL(2)
      GO TO 12
   11 DYDAL(1,I) = 0.0D0
      DYDAL(2,I) = 0.CD0
      Y(I) = C00
   12 CONTINUE
      RETURN
      END


      SUBROUTINE SIMSOL(A,B,N)
C
C
C     THIS SUBROUTINE SOLVES THE LINEAR SIMULTANECUS EQUATION
C     AX = B IN N UNKNOWNS AND RETURNS THE SOLUTIONS IN B
C
C
      IMPLICIT REAL*8(A-H,O-Z)
      DIMENSION A(4,4),B(4)
C
C     CHECK FOR SINGULARITY
C
      DO 11 J=1,N
      JJ = J + 1
      IF(A(J,J)) 7,1,7
    1 IF(J-N) 2,4,4
    2 DO 3 I=JJ,N
      IF(A(I,J)) 5,3,5
    3 CONTINUE
    4 WRITE(3,100)
  100 FORMAT(1X,' NO SOLUTION, SINGULAR MATRIX')
      RETURN
C
C     A(J,J) = 0  INTERCHANGE ROWS
C
    5 DO 6 L=J,N
      T = A(J,L)
      A(J,L) = A(I,L)
    6 A(I,L) = T
      T = B(J)
      B(J) = B(I)
      B(I) = T
C
C     DIVIDE JTH ROW BY A(J,J)
C
    7 T = A(J,J)
      DO 8 KK=J,N
    8 A(J,KK) = A(J,KK)/T
      B(J) = B(J)/T
C
C     GET A(I,J) = 0, I=1,...,J-1,J+1,...,N
C
      DO 11 I=1,N
      IF(I-J) 9,11,9
    9 T = A(I,J)
      DO 10 KK=J,N
   10 A(I,KK) = A(I,KK) - T*A(J,KK)
      B(I) = B(I) - T*B(J)
   11 CONTINUE
      RETURN
      END
```

Sample Output

```
                 DATA TO BE FITTED

           CD                   CL
     0.470000D-01         0.0
     0.500000D-01         0.225000D 00
     0.540000D-01         0.400000D 00
     0.570000D-01         0.500000D 00
     0.600000D-01         0.570000D 00
     0.700000D-01         0.730000D 00
     0.800000D-01         0.830000D 00
     0.100000D 00         0.970000D 00
     0.120000D 00         C.106500D 01
     0.140000D 00         0.113500D 01
     0.162000D 00         0.119000D 01
     0.182000D 00         0.123000D 01
     0.201000D 00         0.126000D 01
     0.226000D 00         0.129000D 01
     0.240000D 00         0.130500D 01
     0.260000D 00         C.132000D 01
     0.280000D 00         0.133500D 01
     0.300000D 00         0.135000D 01
     0.320000D 00         0.136000D 01
```

```
RMS = 0.23450D-01
AL(I) =-0.23614          0.28216          3.0000
RMS = 0.17844D-01
AL(I) =-0.385730D-01     0.985200D-01     3.4610
RMS = 0.13055D-01
AL(I) =-0.514780D-01     0.11010          3.6160
RMS = 0.20523D-01
AL(I) = 0.655460D-01     0.214400D-01     5.1761
RMS = 0.85084D-02
AL(I) = 0.878790D-02     0.449220D-01     5.3366
RMS = 0.70221D-02
AL(I) = 0.407890D-01     0.203320D-01     6.9554
RMS = 0.51926D-02
AL(I) = 0.307680D-01     0.222460D-01     7.1130
RMS = 0.35414D-02
AL(I) = 0.420450D-01     0.134580D-01     8.4475
RMS = 0.43574D-02
AL(I) = 0.463450D-01     0.876130D-02     9.5789
RMS = 0.24494D-02
AL(I) = 0.462370D-01     0.903850D-02     9.6964
RMS = 0.22659D-02
AL(I) = 0.486310D-01     0.740080D-0.     10.303
RMS = 0.22511D-02
AL(I) = 0.494730D-01     0.685760D-02     10.535
RMS = 0.22511D-02
AL(I) = 0.494800D-01     0.685400D-02     10.537
RMS = 0.22511D-02
AL(I) = 0.494800D-01     0.685400D-02     10.537
RMS = 0.22511D-02
AL(I) = 0.494800D-01     0.685400D-02     10.537
RMS = 0.22511D-02
AL(I) = 0.494800D-01     0.685400D-02     10.537
RMS = 0.22511D-02
AL(I) = 0.494800D-01     0.685400D-02     10.537
RMS = 0.22511D-02
AL(I) = 0.494800D-01     0.685400D-02     10.537
RMS = 0.22511D-02
AL(I) = 0.494800D-01     0.685400D-02     10.537
RMS = 0.22511D-02
AL(I) = 0.494800D-01     0.685400D-02     10.537
++++++++++ AFTER 100 ITERATIONS |1.0 - RMS/RMSOLD|, THE RELATIVE ERROR BETWEEN
           TWO SUCCESSIVE VALUES OF RMS, STILL EXCEEDS THE SPECIFIED ERROR PARAMETER = 0.500D-02


                                   RMS = 0.22511D-02
      CD = 0.47000D-01 +  0.494800D-01*CL**2 +  0.685400D-02*CL** 0.10537D 02
```

APPENDIX F - Power Estimation

A step by step procedure for estimating aircraft power available is well outlined in an appendix in Reference 7. While this procedure is very useful in predicting the power available in general, the maximum power available or the maximum power available for continuous operation may be found by using a simplified procedure.

The type of propeller charts given in Figure F-1 has been the standard NACA design chart since 1929. The charts already exist for many propellers; those having R.A.F. 6 and Clark Y airfoil sections can be found in References 7 and 8.

In sizing the propeller for a new design, the first step after choosing a blade section is to calculate C_s from the equation:

$$C_s = \frac{(0.638)(V_D)(\sigma)^{1/5}}{(BHP)^{1/5}\ (N)^{2/5}} \qquad \text{(F-1)}$$

where

V_D = design speed in <u>miles per hour</u>,
$\sigma = \rho/\rho_o$ = density ratio,
BHP = design brake horsepower,
N = design engine revolutions per minute.

Using this value of C_s project upward on Figure F-1 to the broken line of maximum efficiency for C_s. This point determines the design blade angle and a horizontal projection to the V/nD scale gives the design value of V/nD where V = the velocity in <u>feet per second</u>, n = engine revolutions per second, and D = propeller diameter in feet. The blade diameter can then be found by:

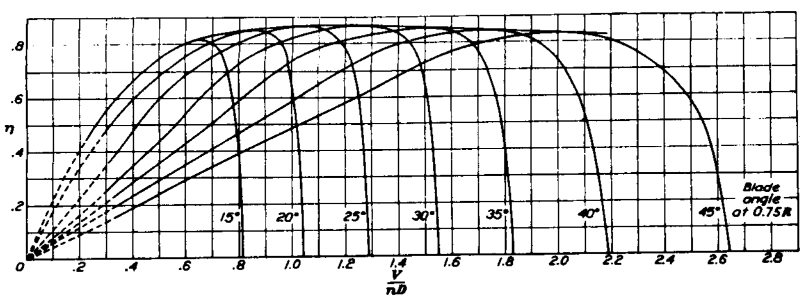
$$D = \frac{(V_D)(88)}{(N)(V/nD)} \quad \text{(in feet)}\ . \qquad \text{(F-2)}$$

The design efficiency is obtained by projecting upward from the design C_s to the efficiency curve for the particular design blade angle.

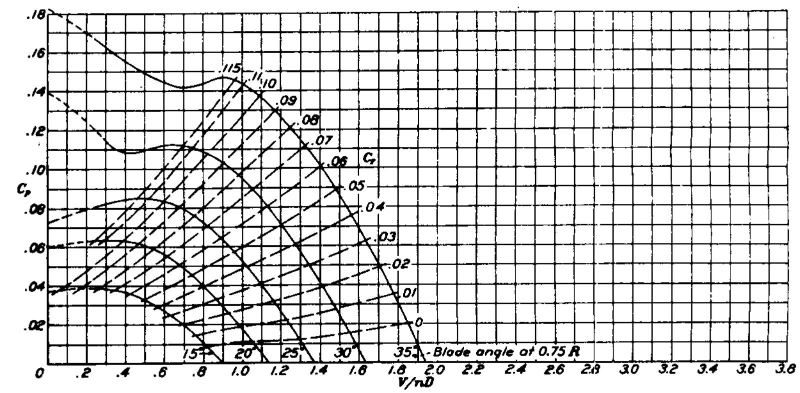
If the aircraft already exists the process of estimating the power available as a function of speed is somewhat different. One already knows the propeller diameter, the propeller section, the engine BHP at rated N, the maximum value of N for continuous operation, the type of propeller (fixed pitch or constant speed), and the blade angle at 75 percent out the propeller radius. The maximum power available as a function of velocity is then calculated according to the procedures described below.

<u>Fixed Pitch Propellers</u>

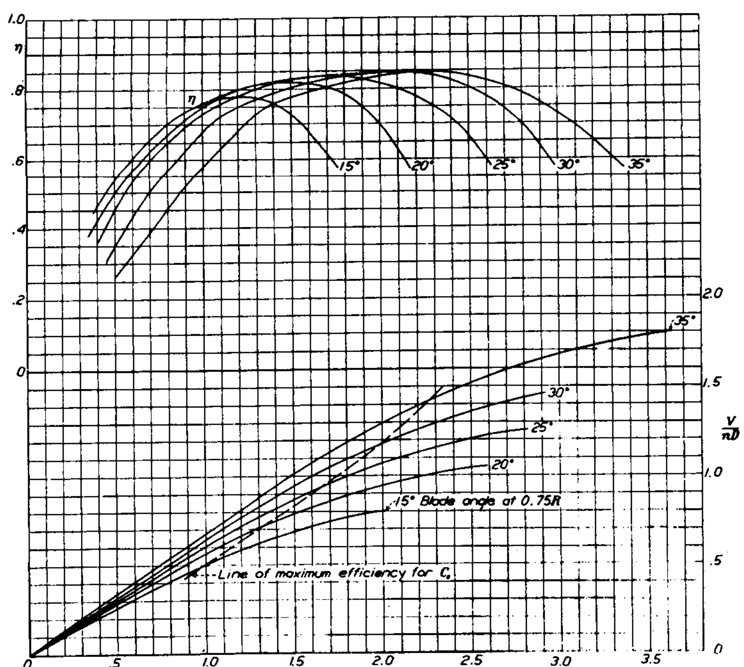
The maximum horsepower for continuous operation at sea level is taken to be:



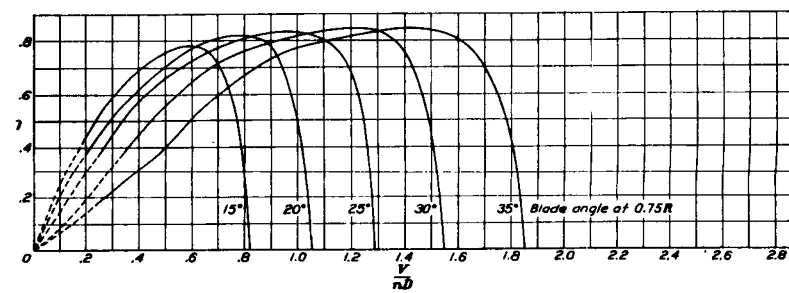
Efficiency curves for propeller 5868-9, Clark Y section, 4 blades.



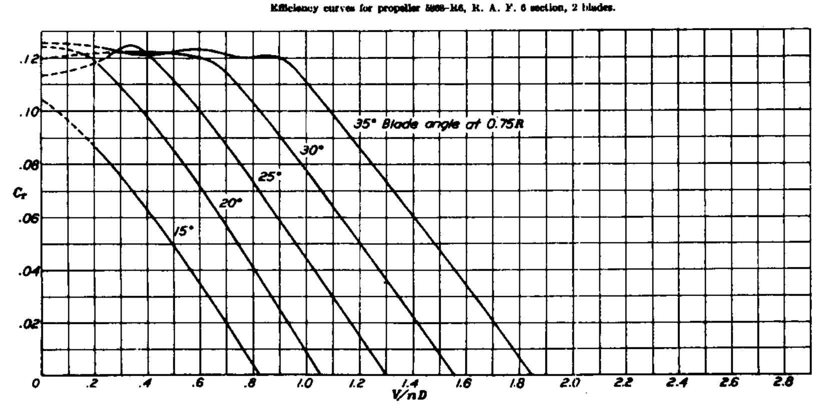
Power-coefficient curves for propeller 5868-R6, R. A. F. 6 section, 2 blades.



Design chart for propeller 5868-R6, R. A. F. 6 section, 2 blades.



Efficiency curves for propeller 5868-R6, R. A. F. 6 section, 2 blades.



Thrust-coefficient curves for propeller 5868-R6, R. A. F. 6 section, 2 blades.

Figure F-1. Example propeller charts for an R.A.F. 6 section.

$$(HP)_{max} = (BHP)(N_c/N_r)(\eta) \quad . \tag{F-3}$$

where

BHP = engine brake horsepower at N_r,
N_c = maximum engine revolutions per minute for continuous operation,
N_r = engine rated revolutions per minute,
η = propeller efficiency.

The propeller efficiency for a given blade angle at 75 percent of the propeller radius is a function only of (V/nD) or V since $n(n = N_c/60)$ and D are specified. Choosing values of (V/nD) and using the efficiency curves for the particular blade section, the value of $(HP)_{max}$ and V can be calculated for each value of (V/nD) chosen. One would thus be able to complete a table similar to the one below.

(V/nD)	$(HP)_{max}$ from Equation (F-3)	V V = (V/nD)(nD)	η (from section chart)
•	•	•	•
•	•	•	•
•	•	•	•
•	•	•	•

Table F-1. Sample tabulation of maximum power and velocity.

Engine efficiency, if known, can be used to multiply the right hand side of Equation (F-3) so as to give a more accurate value of $(HP)_{max}$.

The power available at any altitude is obtained by multiplying the sea level power required by the factor $(\sigma - 0.165)/0.835$ where $\sigma = \rho/\rho_o$ = the ratio of the density of air at altitude to the density of air at sea level (formula obtained from Reference 3).

Constant Speed Propellers

The maximum horsepower for continuous operation at sea level for a constant speed propeller is also given by Equation (F-3). Choosing values of (V/nD) efficiency values can be obtained from the envelope of the efficiency curves since the best efficiency at a particular (V/nD) may be achieved by varying the propeller blade or pitch angle. Since n and D are constant for the maximum power case, one can find n for any value of V. $(HP)_{max}$ for any velocity is then readily determined . The results may be tabulated as in Table F-1.

APPENDIX G - A Predictor Corrector Method for Numerical Integration of Relaxation Differential Equations

By Neill S. Smith

Relaxation differential equations are characterized by a strong dependence of the derivative of the dependent variable on the difference between its own value and a slowly varying function. Conventional Runge-Kutta and predictor-corrector methods are unable to handle these types of equations. Generally, these conventional methods develop strong oscillations when applied to relaxation differential equations.

Trenor (Ref. 31) derived a modification of a fourth-order Runge-Kutta method for use with relaxation differential equations. His method is particularly useful because it becomes identical to the conventional Runge-Kutta in regions where the derivative is not strongly dependent on the value of the dependent variable. Thus his method can be applied to a differential equation which is not of relaxation form over the entire integration range.

Controlling the integration step size in order to maintain a specified accuracy is difficult with Runge-Kutta methods. The usual practice is to integrate from the point x_o to the point $x_o + h$ using the step size h and then integrate again from x_o to $x_o + h$ using the step size h/2. The difference between the two results obtained at $x_o + h$ is used to determine whether the step size should be halved, doubled, or remain the same. This procedure requires eleven evaluations of the derivative function to integrate forward one step.

On the other hand, controlling the step size to maintain a specified accuracy is fairly simple with predictor-corrector methods. The difference between the corrected and predicted values or the difference between two successive interated values of the corrector can be used to control the step size. Thus the predictor-corrector method requires only two or three evaluations of the derivative function per integration step.

Obviously there results a great savings in computational time with predictor-corrector methods; therefore, a predictor-corrector method that could handle relaxation differential equations would be extremely useful. Such a predictor-corrector method is derived in this Appendix by applying Trenor's modification to the corrector of the conventional Adams-Bashforth predictor-corrector method. In regions where the differential equation is not of relaxation form, the modified corrector becomes identical with the conventional Adams-Bashforth corrector.

The procedure for integrating the first-order differential equation

$$y' = \frac{dy}{dx} = f(x,y) \tag{G-1}$$

from the point x_n to $x_{n+1} = x_n + h$ by the conventional Adams-Bashforth method is given below:

(1) An estimate of the value of y at the point x_{n+1} denoted by m_{n+1} is obtained with the predictor equation

$$m_{n+1} = y_n + \frac{h}{24} [55\ y_n' - 37\ y_{n-1}' + 15\ y_{n-2}' - 9\ y_{n-3}'] \ . \tag{G-2}$$

(2) An <u>estimate</u> of the derivative of y at x_{n+1}, denoted by m_{n+1}', is obtained by evaluating Equation (G-1) at the point (x_{n+1}, m_{n+1}).

$$m_{n+1}' = f(x_{n+1}, m_{n+1}) \tag{G-3}$$

(3) Using m_{n+1}' (the estimated derivative of y at x_{n+1}) a final corrected value of y at x_{n+1} is obtained with the corrector equation

$$y_{n+1} = y_n + \frac{h}{24} [9\ m_{n+1}' + 19\ y_n' - 5\ y_{n-1}' + y_{n-2}'] \ . \tag{G-4}$$

Although generally very satisfactory, this method fails when Equation (G-1) is of relaxation form, *i.e.*, it can be written approximately as

$$\frac{dy}{dx} = - P(y - \tilde{y}) \tag{G-5}$$

where P is a large positive number, and $\tilde{y}$ is a slowly varying function of x. A modification of the corrector Equation (G-4) that enables the Adams-Bashforth method to handle the above situation is derived below.

Following Trenor's procedure, it is assumed that Equation (G-1) can be approximated by

$$\frac{dy}{dx} = f(x,y) = - P(y - y_n) + A + B(x - x_n) + C(x - x_n)^2 + D(x - x_n)^3 \tag{G-6}$$

over the interval from $x_{n-3} = x_n - 3h$ to $x_{n+1} = x_n + h$. Equation (G-6) can be integrated to give

$$y_{n+1} = y_n + h\ [AF_1 + hBF_2 + 2h^2CF_3 + 6h^3DF_4] \tag{G-7}$$

where the functions F_n are simple exponential functions of Ph

$$F_o = \exp\ [-Ph]$$

$$F_n = \frac{F_{n-1} - \frac{1}{(n-1)!}}{(-Ph)} = \sum_{k=0}^{\infty} \frac{(-Ph)^k}{(n+k)!} \ . \tag{G-8}$$

The five constants A, B, C, D, and P are evaluated by requiring Equation (G-6) be satisfied at the five points x_{n+i}, $i = 1,0,-1,-2,-3$. The resulting expressions for these five constants are

$$A = y'_n$$

$$hB = \frac{Z_{n-2}}{6} - Z_{n-1} + \frac{Z_n}{2} + \frac{Z_{n+1}}{3}$$

$$2h^2C = Z_{n-1} - 2\,Z_n + Z_{n+1} \tag{G-9}$$

$$6h^3D = -\,Z_{n-2} + 3\,Z_{n-1} - 3\,Z_n + Z_{n+1}$$

$$P = -\,\frac{y'_{n+1} - 4y'_n + 6y'_{n-1} - 4y'_{n-2} + y'_{n-3}}{y_{n+1} - 4y_n + 6y_{n-1} - 4y_{n-2} + y_{n-3}}$$

where $Z_{n+i} = y'_{n+i} + Py_{n+i}$. Since y_{n+1} and y'_{n+1} are used to determine the five constants, Equation (G-7) represents a corrector equation that is to be used in place of the conventional corrector given by Equation (G-4). When $P = 0$ (corresponding to no relaxation-type dependence of y' on y), then $F_1 = 1$, $F_2 = 1/2$, $F_3 = 1/6$ and $F_4 = 1/24$, and Equation (G-7) becomes identical to the conventional corrector given by Equation (G-4). Since P is continually calculated as the integration proceeds, the predictor-corrector method obtained by replacing the corrector given by Equation (G-4) with the modified corrector given by Equation (G-7) will automatically handle any relaxation dependence that appears in the differential equation. In regions where the differential equation has no relaxation dependence ($P = 0$), the method automatically becomes identical to the original predictor-corrector method.

APPENDIX H - A Discussion on Specific Fuel Consumption

Throughout this work it has been assumed that the specific fuel consumption, c, of a piston engine is constant. This of course is a rather gross approximation and it is the purpose of the present section to determine how serious the error is, how one may incorporate a more exact model in the calculations if desired, and how the use of a more exact model will alter the conclusions reached previously.

A piston engine is a very complex machine; thus, unless one uses experimental data giving power and fuel flow rate as functions of manifold pressure and shaft speed in a table look-up form, it is necessary to make some approximations to these characteristics in order to obtain managable functional forms. The experimental characteristics shown in the figures below indicate the magnitude of the problem.

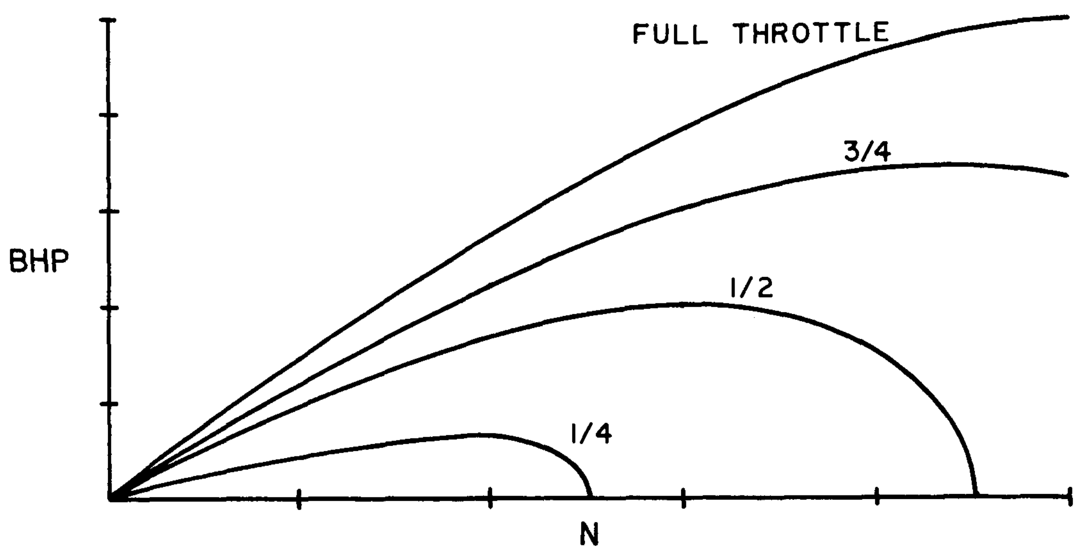


Figure H-1. Typical variation of BHP with RPM for various constant throttle settings.

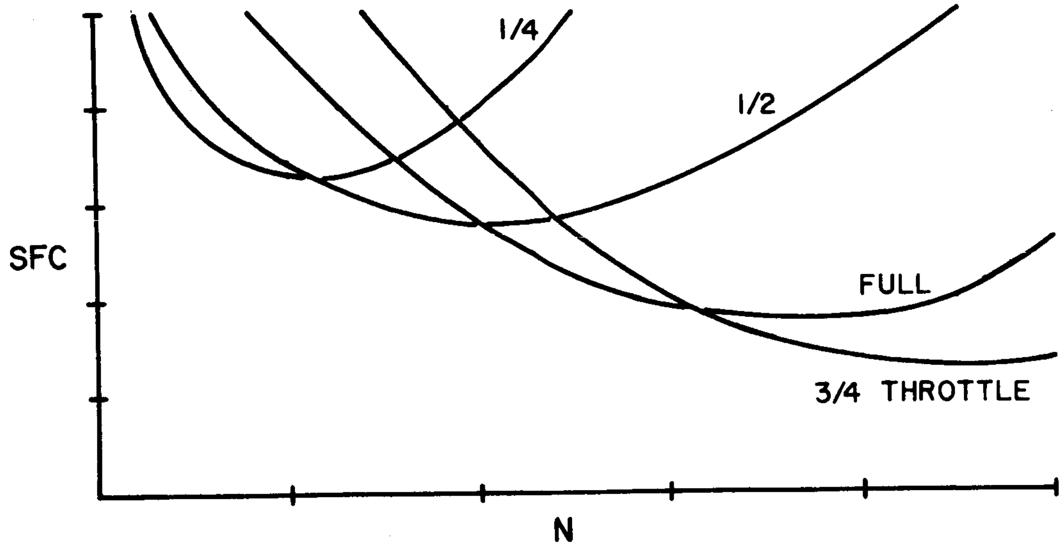


Figure H-2. Typical variation of SFC with RPM for various constant throttle settings.

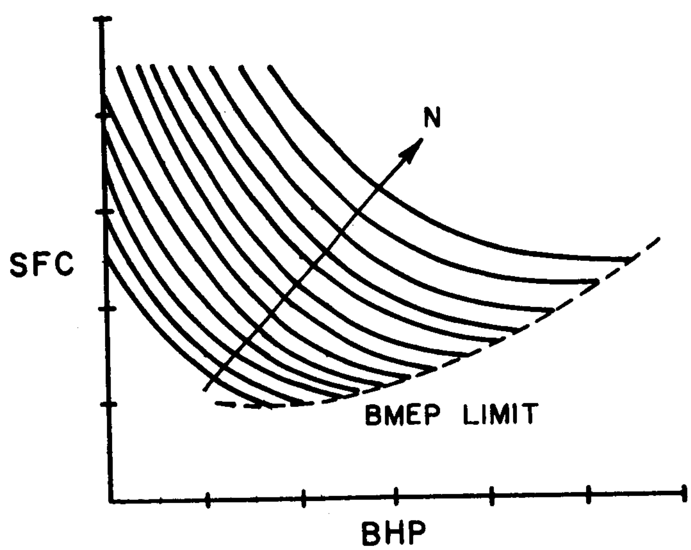


Figure H-3. Typical variation of SFC with BHP for various constant RPM settings.

The constant speed contours in Figure H-3 are given for each 100 RPM above fast idle.

In terms of the throttle position, π, and RPM, N, a fairly good representation of the power output is

$$P = C_1 N \pi^{1/2} \left[1 - \frac{C_2 N^2}{\pi^2} \right]$$

where the value of C_1 and C_2 depends upon the particular engine involved.

During most of the flight regime one could normally attempt to operate at minimum fuel consumption. On an aircraft equipped with a constant speed propeller this means one will generally move down a constant N contour (Figure H-3) until full throttle is reached and then along a constant manifold pressure contour. This type of operation can usually be represented fairly accurately by a function of the type

$$c = \frac{A_1}{P} + A_2 P^3 \quad .$$

Obviously, more complex functions can be used to obtain more accurate descriptions of experimental results.

For the moment suppose the foregoing expression for c represents the physical situation adequately. How then does one incorporate it in the performance analysis, and how are the previous conclusions one obtained on the basis of the simple, linear fuel-flow-power function altered? For the more general fuel consumption function,

$$\dot{W} = A_1 + A_2 P^4 \quad .$$

Mechanically, such a function presents no substantial difficulties in the computation procedure, although the program instructions would have to be changed to accomodate it. Generally, it is to be expected that such a fuel-flow-power relation does not alter the speed for maximum endurance and reduces the range achievable with high power (>75%) trajectories.

If one wished to perform the computation precisely, the following procedure is suggested:

1. Obtain from the engine manufacturer the most reliable data he has on the engine in question and plot the following curves: c versus P with N as parameter, c versus P with manifold pressure as parameter. The plots should include the effects of a particular propeller's efficiency so that power is really thrust horsepower.

2. Employ a least squares polynomial fit to each family of curves.

3. Then when one specifies a manifold pressure and an engine speed, he solves the two equations simultaneously to obtain c and P.

4. The path performance computation is changed as follows:

(a) Either manifold pressure or engine speed or both must be specified. If both are specified c and P are both found and W can be computed. If only one is specified the power required which comes from a solution of the system of equations in Appendix B must be used in conjunction with the two curve fits to find c and engine speed or manifold pressure, whichever was previously unknown.

(b) The value of c corresponding to the required value of P is then inserted in the path performance equations and the computation continues for another small increment in time.

The magnitude of the programming task is quite evident. Unfortunately, the pressures of time did not permit its completion during the preparation of the present work.

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